7.2: Trigonometric Integrals

Learning Objectives

- Solve integration problems involving products and powers of \(\sin x\) and \(\cos x\).
- Solve integration problems involving products and powers of \(\tan x\) and \(\sec x\).
- Use reduction formulas to solve trigonometric integrals.

In this section we look at how to integrate a variety of products of trigonometric functions. These integrals are called trigonometric integrals. They are an important part of the integration technique called trigonometric substitution, which is featured in Trigonometric Substitution. This technique allows us to convert algebraic expressions that we may not be able to integrate into expressions involving trigonometric functions, which we may be able to integrate using the techniques described in this section. In addition, these types of integrals appear frequently when we study polar, cylindrical, and spherical coordinate systems later. Let’s begin our study with products of \(\sin x\) and \(\cos x\).

Integrating Products and Powers of \(\sin x\) and \(\cos x\)

A key idea behind the strategy used to integrate combinations of products and powers of \(\sin x\) and \(\cos x\) involves rewriting these expressions as sums and differences of integrals of the form \(\int \sin^j x \cos x \, dx\) or \(\int \cos^j x \sin x \, dx\). After rewriting these integrals, we evaluate them using \(u\)-substitution. Before describing the general process in detail, let’s take a look at the following examples.

Example 1: Integrating \(\int \cos^3 x \sin x \, dx\)

Evaluate \(\int \cos^3 x \sin x \, dx\).
Solution

Use \(u\)-substitution and let \(u=\cos x\). In this case, \(du=-\sin x\,dx\).

Thus,

\[
\int \cos^3 x \sin x \, dx = -\frac{1}{4}u^4 + C = -\frac{1}{4}\cos^4 x + C.
\]

Exercise (PageIndex(1))

Evaluate \(\int \sin^4 x \cos x \, dx\).

Hint

Let \(u=\sin x\).

Answer

\(\int \sin^4 x \cos x \, dx = \frac{1}{5}\sin^5 x + C\)

Example (PageIndex(2)): A Preliminary Example: Integrating \(\int \cos^j x \sin^k x \, dx\) where \(k\) is Odd

Evaluate \(\int \cos^2 x \sin^3 x \, dx\).

Solution

To convert this integral to integrals of the form \(\int \cos^j x \sin x \, dx\), rewrite \(\sin^3 x = \sin^2 x \sin x\) and make the substitution \(\sin^2 x = 1 - \cos^2 x\).

Thus,

\[
\begin{align*}
\int \cos^2 x \sin^3 x \, dx &= \int (1 - \cos^2 x) \sin x \, dx \quad \text{Let } u = \cos x; \quad \text{then } \, du = -\sin x \, dx. \\
&= -\int u^2 (1 - u^2) \, du \\
&= -\frac{1}{3}u^3 + C \\
&= -\frac{1}{3}\cos^3 x + C.
\end{align*}
\]

Exercise (PageIndex(2))

Evaluate \(\int \cos^3 x \sin^2 x \, dx\).

Hint

Write \(\cos^3 x = \cos^2 x \cos x = (1 - \sin^2 x) \cos x\) and let \(u = \sin x\).
In the next example, we see the strategy that must be applied when there are only even powers of \( \sin x \) and \( \cos x \). For integrals of this type, the identities

\[
\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos(2x) = \frac{1 - \cos(2x)}{2}
\]

and

\[
\cos^2 x = \frac{1}{2} + \frac{1}{2} \cos(2x) = \frac{1 + \cos(2x)}{2}
\]

are invaluable. These identities are sometimes known as **power-reducing identities** and they may be derived from the double-angle identity \( \cos(2x) = \cos^2 x - \sin^2 x \) and the Pythagorean identity \( \cos^2 x + \sin^2 x = 1 \).

Example \( \PageIndex{3} \): Integrating an Even Power of \( \sin x \)

Evaluate \( \displaystyle \int \sin^2 x \, dx \).

**Solution**

To evaluate this integral, let’s use the trigonometric identity \( \sin^2 x = \frac{1}{2} - \frac{1}{2} \cos(2x) \). Thus,

\[
\int \sin^2 x \, dx = \int \left( \frac{1}{2} - \frac{1}{2} \cos(2x) \right) \, dx = \frac{1}{2} x - \frac{1}{4} \sin(2x) + C.
\]

Exercise \( \PageIndex{3} \)

Evaluate \( \displaystyle \int \cos^2 x \, dx \).

**Hint**

\( \cos^2 x = \frac{1}{2} + \frac{1}{2} \cos(2x) \)

**Answer**

\[
\int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{4} \sin(2x) + C
\]

The general process for integrating products of powers of \( \sin x \) and \( \cos x \) is summarized in the following set of guidelines.

**Problem-Solving Strategy: Integrating Products and Powers of \( \sin x \) and \( \cos x \)**

To integrate \( \int \cos^j x \sin^k x \, dx \) use the following strategies:

1. If \( k \) is odd, rewrite \( \sin^k x = \sin^{k-1} x \sin x \) and use the identity \( \sin^2 x = 1 - \cos^2 x \) to rewrite
\(\sin^{k-1}x\) in terms of \(\cos x\). Integrate using the substitution \(u=\cos x\). This substitution makes \(du=-\sin x\,dx\).

2. If \(j\) is odd, rewrite \(\cos^jx=\cos^{j-1}x\cos x\) and use the identity \(\cos^2x=1-\sin^2x\) to rewrite \(\cos^{j-1}x\) in terms of \(\sin x\). Integrate using the substitution \(u=\sin x\). This substitution makes \(du=\cos x\,dx\). (Note: If both \(j\) and \(k\) are odd, either strategy 1 or strategy 2 may be used.)

3. If both \(j\) and \(k\) are even, use \(\sin^2x=(1/2)-(1/2)\cos(2x)\) and \(\cos^2x=(1/2)+(1/2)\cos(2x)\). After applying these formulas, simplify and reapply strategies 1 through 3 as appropriate.

Example \(\PageIndex{4}\): Integrating \(\int \cos^jx\sin^kx\,dx\) where \(k\) is Odd

Evaluate \(\int \cos^8x\sin^5x\,dx\).

Solution

Since the power on \(\sin x\) is odd, use strategy 1. Thus,

\[
\begin{align*}
\int \cos^8x\sin^5x\,dx &= \int \cos^8x\sin^4x\sin x\,dx & \text{Break off } \sin x. \\
&= \int \cos^8x(\sin^2x)^2\sin x\,dx & \text{Rewrite } \sin^4x=(\sin^2x)^2. \\
&= \int \cos^8x(1-\cos^2x)^2\sin x\,dx & \text{Substitute } \sin^2x=1-\cos^2x. \\
&= \int \cos^8x(-u^2)^2(-du) & \text{Let } u=\cos x\text{ and } du=-\sin x\,dx. \\
&= \int (-u^8+2u^{10}-u^{12})\,du & \text{Expand.} \\
&= -\frac{1}{9}u^9+\frac{2}{11}u^{11}-\frac{1}{13}u^{13}+C & \text{Evaluate the integral.} \\
&= -\frac{1}{9}\cos^9x+\frac{2}{11}\cos^{11}x-\frac{1}{13}\cos^{13}x+C & \text{Substitute } u=\cos x. 
\end{align*}
\]

Example \(\PageIndex{5}\): Integrating \(\int \cos^jx\sin^kx\,dx\) where \(k\) and \(j\) are Even

Evaluate \(\int \sin^4x\,dx\).

Solution: Since the power on \(\sin x\) is even \((k=4)\) and the power on \(\cos x\) is even \((j=0)\), we must use strategy 3. Thus,

\[\int \sin^4x\,dx=\int (\sin^2x)^2\,dx\] Rewrite \(\sin^4x=(\sin^2x)^2\).

\[
\begin{align*}
\int (\frac{1}{2} - \frac{1}{2}\cos(2x))^2\,dx &= \int (\frac{1}{2} - \frac{1}{2}\cos(2x))^2\,dx & \text{Substitute } \sin^2x=\frac{1}{2} - \frac{1}{2}\cos(2x). \\
&= \int (\frac{1}{2} - \frac{1}{2}\cos(2x))\,dx & \text{Expand } (\frac{1}{2} - \frac{1}{2}\cos(2x))^2. \\
&= \int (\frac{3}{8} - \frac{1}{2}\cos(2x)+\frac{1}{8}\cos(4x))\,dx & \text{Expand } \frac{1}{2} - \frac{1}{2}\cos(2x). \\
&= \int \frac{3}{8}\,dx - \frac{1}{2}\int \cos(2x)\,dx + \frac{1}{8}\int \cos(4x)\,dx. \\
\end{align*}
\]

Since \(\cos(2x)\) has an even power, substitute \(\cos^2(2x)=\frac{1}{2}+\frac{1}{2}\cos(4x)\):

\[
\begin{align*}
\int (\frac{3}{8} - \frac{1}{2}\cos(2x)+\frac{1}{8}\cos(4x))\,dx &= \frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{16}\sin(4x) + C. \\
\end{align*}
\] Simplify.
Exercise $\PageIndex{4}$

Evaluate $\displaystyle \int \cos^3 x \, dx$.

**Hint**

Use strategy 2. Write $(\cos^3 x = \cos^2 x \cos x)$ and substitute $(\cos^2 x = 1 - \sin^2 x)$.

**Answer**

$\displaystyle \int \cos^3 x \, dx = \sin x - \frac{1}{3} \sin^3 x + C$

Exercise $\PageIndex{5}$

Evaluate $\displaystyle \int \cos^2(3x) \, dx$.

**Hint**

Use strategy 3. Substitute $(\cos^2(3x) = \frac{1}{2} + \frac{1}{2} \cos(6x))$.

**Answer**

$\displaystyle \int \cos^2(3x) \, dx = \frac{1}{2} x + \frac{1}{12} \sin(6x) + C$

In some areas of physics, such as quantum mechanics, signal processing, and the computation of Fourier series, it is often necessary to integrate products that include $\sin(ax)$, $\sin(bx)$, $\cos(ax)$, and $\cos(bx)$. These integrals are evaluated by applying trigonometric identities, as outlined in the following rule.

**Rule: Integrating Products of Sines and Cosines of Different Angles**

To integrate products involving $\sin(ax)$, $\sin(bx)$, $\cos(ax)$, and $\cos(bx)$, use the substitutions

\[
\sin(ax) \sin(bx) = \frac{1}{2} \cos((a - b)x) - \frac{1}{2} \cos((a + b)x)
\]

\[
\sin(ax) \cos(bx) = \frac{1}{2} \sin((a - b)x) + \frac{1}{2} \sin((a + b)x)
\]

\[
\cos(ax) \cos(bx) = \frac{1}{2} \cos((a - b)x) + \frac{1}{2} \cos((a + b)x)
\]

These formulas may be derived from the sum-of-angle formulas for sine and cosine.

Example $\PageIndex{6}$: Evaluating $\int \sin(5x) \cos(3x) \, dx$

Evaluate $\displaystyle \int \sin(5x) \cos(3x) \, dx$.
Solution: Apply the identity \(\sin(5x)\cos(3x)=\frac{1}{2}\sin(2x)+\frac{1}{2}\sin(8x)\). Thus,
\[
\int \sin(5x)\cos(3x)\,dx = \int \frac{1}{2}\sin(2x)+\frac{1}{2}\sin(8x)\,dx = -\frac{1}{4}\cos(2x)-\frac{1}{16}\cos(8x)+C.
\]

Exercise \(\PageIndex{6}\)
Evaluate \(\int \cos(6x)\cos(5x)\,dx\).

Hint
Substitute \(\cos(6x)\cos(5x)=\frac{1}{2}\cos x+\frac{1}{2}\cos(11x)\).

Answer
\[
\int \cos(6x)\cos(5x)\,dx = \frac{1}{2}\sin x+\frac{1}{22}\sin(11x)+C
\]

Integrating Products and Powers of \((\tan x)\) and \((\sec x)\)

Before discussing the integration of products and powers of \((\tan x)\) and \((\sec x)\), it is useful to recall the integrals involving \((\tan x)\) and \((\sec x)\) we have already learned:

1. \(\int \sec^2 x\,dx = \tan x+C\)
2. \(\int \sec x \tan x\,dx = \sec x+C\)
3. \(\int \tan x\,dx = \ln|\sec x|+C\)
4. \(\int \sec x\,dx = \ln|\sec x+\tan x|+C\).

For most integrals of products and powers of \((\tan x)\) and \((\sec x)\), we rewrite the expression we wish to integrate as the sum or difference of integrals of the form \(\int \tan^j x \sec^2 x\,dx\) or \(\int \sec^j x \tan x\,dx\). As we see in the following example, we can evaluate these new integrals by using u-substitution.

Example \(\PageIndex{7}\): Evaluating \(\int \sec^5 x \tan x\,dx\)

Evaluate \(\int \sec^5 x \tan x\,dx\).

Solution: Start by rewriting \(\int \sec^5 x \tan x\,dx\) as \(\int \sec^4 x \sec x \tan x\,dx\).
\[
\begin{align*}
\int \sec^5 x \tan x\,dx &= \int \sec^4 x \sec x \tan x\,dx & & \text{Let } u=\sec x; \text{then}, \, du=\sec x \tan x \,dx. \\
&= \int u^4\,du & & \text{Evaluate the integral.} \\
&= \frac{1}{5}u^5+C & & \text{Substitute } u=\sec^5 x.
\end{align*}
\]
Exercise \(\PageIndex{7}\)

Evaluate \(\int \tan^5x \sec^2x \, dx\).

Hint

Let \((u=\tan x)\) and \((du=\sec^2 x \, dx)\)

\[\int \tan^5x \sec^2x \, dx = \frac{1}{6} \tan^6x + C\]

Problem-Solving Strategy: Integrating \(\int \tan^kx \sec^jx \, dx\)

To integrate \(\int \tan^kx \sec^jx \, dx\) use the following strategies:

1. If \((j)\) is even and \((j\geq 2)\) rewrite \((\sec^jx=\sec^{j-2}x \sec^2x)\) and use \((\sec^2x=\tan^2x+1)\) to rewrite \((\sec^{j-2}x)\) in terms of \((\tan x)\). Let \((u=\tan x)\) and \((du=\sec^2x \, dx)\)

2. If \((k)\) is odd and \((j\geq 1)\), rewrite \((\tan^kx \sec^jx=\tan^{k-1}x \sec^{j-1}x \sec x \tan x)\) and use \((\tan^2x=\sec^2x-1)\) to rewrite \((\tan^{k-1}x)\) in terms of \((\sec x)\). Let \((u=\sec x)\) and \((du=\sec x \tan x \, dx)\) (Note: If \((j)\) is even and \((k)\) is odd, then either strategy 1 or strategy 2 may be used.)

3. If \((k)\) is odd where \((k\geq 3)\) and \((j=0)\), rewrite \((\tan^kx=\tan^{k-2}x \tan^2x)\) \(\tan^2x=\tan^{k-2}x(\sec^2x-1)=\tan^{k-2}x \sec^2x-\tan^{k-2}x\). It may be necessary to repeat this process on the \((\tan^{k-2}x)\) term.

4. If \((k)\) is even and \((j)\) is odd, then use \((\tan^2x=\sec^2x-1)\) to express \((\tan^kx)\) in terms of \((\sec x)\). Use integration by parts to integrate odd powers of \((\sec x)\)

Example \(\PageIndex{8}\): Integrating \(\int \tan^kx \sec^jx \, dx\) when \((j)\) is Even

Evaluate \(\int \tan^6x \sec^4x \, dx\)

Solution

Since the power on \((\sec x)\) is even, rewrite \((\sec^4x=\sec^2x \sec^2x)\) and use \((\sec^2x=\tan^2x+1)\) to rewrite the first \((\sec^2x)\) in terms of \((\tan x)\). Thus,

\[\int \tan^6x \sec^4x \, dx=\int \tan^6x(\tan^2x+1) \sec^2x \, dx\]

Let \((u=\tan x)\) and \((du=\sec^2x \, dx)\)
\( \int (u^6(u^2+1)) \, du \) Expand.

\( \int (u^8+u^6) \, du \) Evaluate the integral.

\( \int \left( \frac{1}{9}u^9 + \frac{1}{7}u^7 + C \right) \) Substitute \( \tan x = u \).

\( \int \left( \frac{1}{9}\tan^9 x + \frac{1}{7}\tan^7 x + C \right) \)

Example \( \PageIndex{9} \): Integrating \( \int \tan^k x \sec^j x \, dx \) when \( k \) is Odd

Evaluate \( \int \tan^5 x \sec^3 x \, dx \)

Solution

Since the power on \( \tan x \) is odd, begin by rewriting \( \tan^5 x \sec^3 x = \tan^4 x \sec^2 x \sec x \tan x \) Thus,

\( \int \tan^5 x \sec^3 x \, dx = \int \tan^4 x \sec^2 x \sec x \tan x \, dx \) Write \( \int \tan^4 x = (\tan^2 x)^2 \).

\( \int \tan^5 x \sec^3 x \, dx = \int (\sec^2 x - 1)^2 \sec^2 x \sec x \tan x \, dx \) Use \( \int \tan^2 x = \sec^2 x - 1 \).

\( \int (u^2 - 1)^2 u^2 du \) Expand.

\( \int (u^6 - 2u^4 + u^2) du \) Integrate.

\( \int \left( \frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3 + C \right) \) Substitute \( \sec x = u \).

\( \int \left( \frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C \right) \)

Example \( \PageIndex{10} \): Integrating \( \int \tan^k x \, dx \) where \( k \) is Odd and \( k \geq 3 \)

Evaluate \( \int \tan^3 x \, dx \)

Solution

Begin by rewriting \( \int \tan^3 x \, dx = \tan x \tan^2 x = \tan x (\sec^2 x - 1) = \tan x \sec^2 x - \tan x \) Thus,

\( \int \tan^3 x \, dx = \int (\tan x \sec^2 x - \tan x) \, dx \)

\( \int \tan x \sec^2 x \, dx = \int \tan x \, dx \)

\( \int \sec^2 x \, dx = \tan x + C \)

For the first integral, use the substitution \( u = \tan x \). For the second integral, use the formula.
Example (PageIndex{11}): Integrating \(\int \sec^3 x \, dx\)

Integrate \(\int \sec^3 x \, dx\).

Solution

This integral requires integration by parts. To begin, let \((u=\sec x)\) and \((dv=\sec^2 x)\). These choices make \((du=\sec x \tan x)\) and \((v=\tan x)\). Thus,

\[
\int \sec^3 x \, dx = \sec x \tan x - \int \tan x \sec x \tan x \, dx
\]

Simplify.

\[
= \sec x \tan x - \int (\sec^2 x - 1) \sec x \, dx
\]

Substitute \((\tan^2 x = \sec^2 x - 1)\).

\[
= \sec x \tan x + \int \sec x \, dx - \int \sec^3 x \, dx
\]

Rewrite.

\[
= \sec x \tan x + \ln|\sec x + \tan x| - \int \sec^3 x \, dx
\]

Evaluate \(\int \sec x \, dx\).

We now have

\[
\int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x| - \int \sec^3 x \, dx.
\]

Since the integral \(\int \sec^3 x \, dx\) has reappeared on the right-hand side, we can solve for \(\int \sec^3 x \, dx\) by adding it to both sides. In doing so, we obtain

\[
2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x|.
\]

Dividing by 2, we arrive at

\[
\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C
\]

Exercise (PageIndex{8})

Evaluate \(\int \tan^3 x \sec^7 x \, dx\).

Hint

Use Example (PageIndex{9}) as a guide.

Answer

\(\int \tan^3 x \sec^7 x \, dx = \frac{1}{9} \sec^9 x - \frac{1}{7} \sec^7 x + C\)
Reduction Formulas

Evaluating \( \int \sec^n x \, dx \) for values of \( n \) where \( n \) is odd requires integration by parts. In addition, we must also know the value of \( \int \sec^{n-2} x \, dx \) to evaluate \( \int \sec^n x \, dx \). The evaluation of \( \int \tan^n x \, dx \) also requires being able to integrate \( \int \tan^{n-2} x \, dx \). To make the process easier, we can derive and apply the following power reduction formulas. These rules allow us to replace the integral of a power of \( \sec x \) or \( \tan x \) with the integral of a lower power of \( \sec x \) or \( \tan x \).

Rule: Reduction Formulas for \( \int \sec^n x \, dx \) and \( \int \tan^n x \, dx \)

\[
\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx
\]

\[
\int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, dx
\]

The first power reduction rule may be verified by applying integration by parts. The second may be verified by following the strategy outlined for integrating odd powers of \( \tan x \).

Example \( \PageIndex{12} \): Revisiting \( \int \sec^3 x \, dx \)

Apply a reduction formula to evaluate \( \int \sec^3 x \, dx \).

Solution: By applying the first reduction formula, we obtain

\[
\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \int \sec x \, dx
\]

\[
= \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C.
\]

Example \( \PageIndex{13} \): Using a Reduction Formula

Evaluate \( \int \tan^4 x \, dx \).

Solution: Applying the reduction formula for \( \int \tan^4 x \, dx \) we have

\[
\int \tan^4 x \, dx = \frac{1}{3} \tan^3 x - \int \tan^2 x \, dx
\]

\[
= \frac{1}{3} \tan^3 x - \left( \tan x - \int \tan^0 x \, dx \right)
\]

Apply the reduction formula to \( \int \tan^2 x \, dx \).

\[
= \frac{1}{3} \tan^3 x - \tan x + \int 1 \, dx
\]

Simplify.

\[
= \frac{1}{3} \tan^3 x - \tan x + x + C
\]

Evaluate \( \int 1 \, dx \).

Exercise \( \PageIndex{9} \)

Apply the reduction formula to \( \int \sec^5 x \, dx \).
Hint

Use reduction formula 1 and let \( n = 5 \).

Answer

\[
\int \sec^5 x \, dx = \frac{1}{4} \sec^3 x \tan x + \frac{3}{4} \int \sec^3 x \, dx
\]

Key Concepts

- Integrals of trigonometric functions can be evaluated by the use of various strategies. These strategies include
  1. Applying trigonometric identities to rewrite the integral so that it may be evaluated by \((u\)-substitution
  2. Using integration by parts
  3. Applying trigonometric identities to rewrite products of sines and cosines with different arguments as the sum of individual sine and cosine functions
  4. Applying reduction formulas

Key Equations

To integrate products involving \(\sin(ax), \sin(bx), \cos(ax),\) and \(\cos(bx),\) use the substitutions.

- Sine Products
  \[
  \sin(ax) \sin(bx) = \frac{1}{2} \cos((a-b)x) - \frac{1}{2} \cos((a+b)x)
  \]

- Sine and Cosine Products
  \[
  \sin(ax) \cos(bx) = \frac{1}{2} \sin((a-b)x) + \frac{1}{2} \sin((a+b)x)
  \]

- Cosine Products
  \[
  \cos(ax) \cos(bx) = \frac{1}{2} \cos((a-b)x) + \frac{1}{2} \cos((a+b)x)
  \]

- Power Reduction Formula
  \[
  \int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx
  \]

- Power Reduction Formula
  \[
  \int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-2} x + \frac{1}{n-1} \int \tan^{n-2} x \, dx
  \]
Glossary

**power reduction formula**
a rule that allows an integral of a power of a trigonometric function to be exchanged for an integral involving a lower power

**trigonometric integral**
an integral involving powers and products of trigonometric functions

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