The Meaning of Definite Integrals of Vector-Valued Functions

Now let's turn our attention to the meaning of a definite integral of a vector-valued function. The context in which this will make the most sense is where the function we integrate is a velocity function. That is,

\[ \int_a^b \vecs v(t) \, dt \]

We know that the antiderivative of velocity is position and that this definite integral gives us the change in position over the time interval, \((a \le t \le b)\). In other words,

\[ \int_a^b \vecs v(t) \, dt = \vecs r(b) - \vecs r(a) \]

Thus, the definite integral of velocity over a time interval \((a \le t \le b)\) gives us the displacement vector that indicates the change in position over this time interval.

In general, the definite integral

\[ \int_a^b \vecs r(t) \, dt = \vecs q(b) - \vecs q(a) \]

where \(\vecs q(t)\) is the antiderivative of \(\vecs r(t)\), gives us a change in the antiderivative of our vector-valued function over the given interval \([a,b]\). This will always be a constant vector that would fit from tip to tip of the vectors given by the antiderivative function at \(t = a\) and \(t = b\), respectively (assuming the vectors were placed in standard position).