# 4.5 E: Optimization Exercises

## 4.5: Applied Optimization Problems

For the following exercises, answer by proof, counterexample, or explanation.

311) When you find the maximum for an optimization problem, why do you need to check the sign of the derivative around the critical points?

**Answer:**

The critical points can be the minima, maxima, or neither, so you need check.

312) Why do you need to check the endpoints for optimization problems?

313) True or False. For every continuous nonlinear function, you can find the value $x$ that maximizes the function.

**Answer:**

False; $y=-x^2$ has a minimum only

314) True or False. For every continuous nonconstant function on a closed, finite domain, there exists at least one $x$ that minimizes or maximizes the function.

For the following exercises, set up and evaluate each optimization problem.

315) To carry a suitcase on an airplane, the length $+width+$ height of the box must be less than or equal to $62in$. (a) Assuming the height is fixed, show that the maximum volume is $V=h(31-(\frac{1}{2})h)^2.$ (b) What height allows you to have the largest volume?

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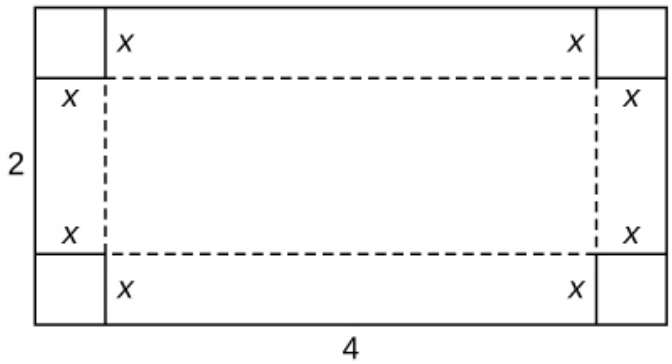
**Answer:**

(a) substitute length = 62 - width - height into Volume to get $V(w)$; consider height a constant and set $V'(w)=0.$ Solve for $w$ & show it's a max. Then rewrite Volume function with this value for $w$ (b) $h=\frac{62}{3}$ in.

316) You are constructing a cardboard box from a flat piece of cardboard with dimensions $2\ m$ by $4\ m.$ You then cut equal-size squares from each corner so you may fold the edges. What are the dimensions of the box with the largest volume?




317) Find the positive integer that minimizes the sum of the number and its reciprocal.

**Answer:**

When $1$

318) Find two positive integers such that their sum is $10$, and minimize and maximize the sum of their squares.

For the following exercises, consider the construction of a pen to enclose an area.

319) You have $400ft$ of fencing to construct a rectangular pen for cattle. What are the dimensions of the pen that maximize the area?

**Answer:**

$100ft\ by\ 100ft$

320) You have $800ft$ of fencing to make a pen for hogs. If you have a river on one side of your property, what is the dimension of the rectangular pen that maximizes the area?

321) You need to construct a fence around an area of $1600ft.$ What are the dimensions of the rectangular pen to minimize the amount of material needed?

**Answer:**

$40ft\ by\ 40ft$

322) Two poles are connected by a wire that is also connected to the ground. The first pole is $20ft$ tall and the second pole is $10ft$ tall. There is a distance of $30ft$ between the two poles. Where should the wire be anchored to the ground to minimize the amount of wire needed?

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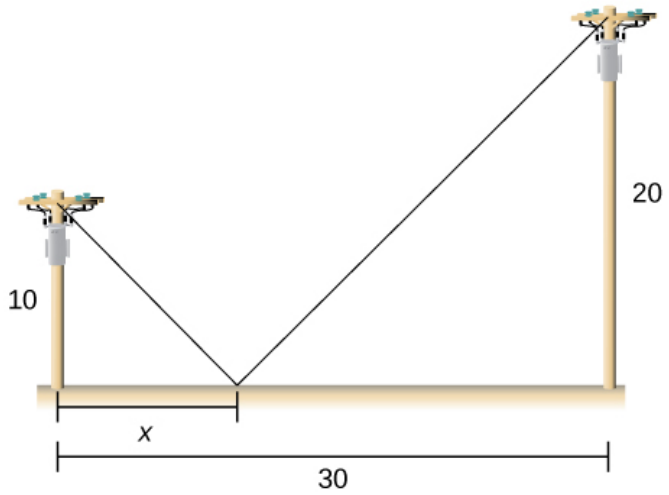
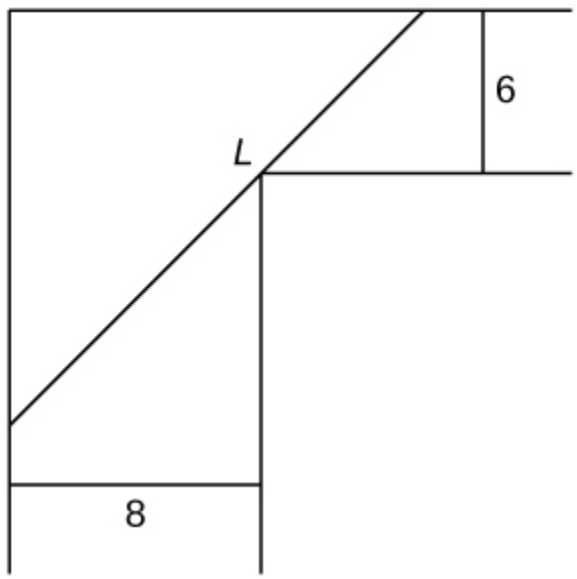



323) [T] You are moving into a new apartment and notice there is a corner where the hallway narrows from \(8 ft to 6 ft\). What is the length of the longest item that can be carried horizontally around the corner?




**Answer:**

19.73 ft

324) A patient's pulse measures \(70 bpm, 80 bpm\), then \(120 bpm.\) To determine an accurate measurement of pulse, the doctor wants to know what value minimizes the expression \((x−70)^2+(x−80)^2+(x−120)^2\)? What value minimizes it?

325) In the previous problem, assume the patient was nervous during the third measurement, so we only weight that value half as much as the others. What is the value that minimizes \((x−70)^2+ (x−80)^2+\frac{1}{2}(x−120)^2?\)

**Answer:**

\(84 bpm\)

326) You can run at a speed of \(6\) mph and swim at a speed of \(3\) mph and are located on the shore, \(4\) miles east of an

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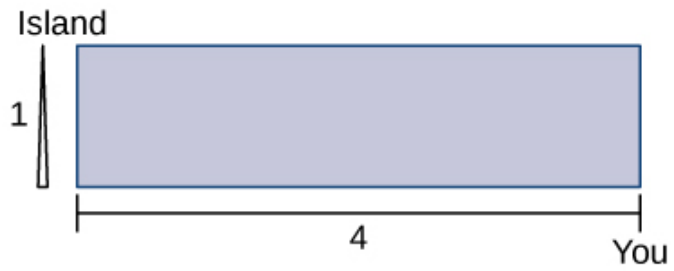
island that is $1$ mile north of the shoreline. How far should you run west to minimize the time needed to reach the island?

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329) A truck uses gas as $g(v)=av+\frac{b}{v}$, where $v$ represents the speed of the truck and $g$ represents the gallons of fuel per mile. At what speed is fuel consumption minimized?

**Answer:**
$v=\sqrt{\frac{b}{a}}$

For the following exercises, consider a limousine that gets $m(v)=\frac{(120-2v)}{5}mi/gal$ at speed $v$, the chauffeur costs $\$15/h$, and gas is $\$3.5/gal.$

330) Find the cost per mile at speed $v.$

331) Find the cheapest driving speed.

**Answer:**
approximately $34.02mph$

For the following exercises, consider a pizzeria that sell pizzas for a revenue of $R(x)=ax$ and costs $C(x)=b+cx+dx^2$, where $x$ represents the number of pizzas.

332) Find the profit function for the number of pizzas. How many pizzas gives the largest profit per pizza?

333) Assume that $R(x)=10x$ and $C(x)=2x+x^2$. How many pizzas sold maximizes the profit?

**Answer:**
$4$

334) Assume that $R(x)=15x,$ and $C(x)=60+3x+\frac{1}{2}x^2$. How many pizzas sold maximizes the profit?

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For the following exercises, consider a wire $4ft$ long cut into two pieces. One piece forms a circle with radius r and the other forms a square of side $x$.

335) Choose $x$ to maximize the sum of their areas.

**Answer:**
$0$; if you got a nice critical value, it does NOT test out to be a maximum. Use it for problem #336.

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336) Choose $x$ to minimize the sum of their areas.

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For the following exercises, consider two nonnegative numbers $x$ and $y$ such that $x+y=10$. Maximize and minimize the quantities.

337) $xy$

**Answer:**
Maximal: $x=5,y=5;$ minimal: $x=0,y=10$ and $y=0,x=10$

338 $x^2y^2$

339) $y-\frac{1}{x}$

**Answer:**
Maximal: $x=1,y=9;$ minimal: none

340) $x^2-y$

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For the following exercises, draw the given optimization problem and solve.

341) Find the volume of the largest right circular cylinder that fits in a sphere of radius $1$.

**Answer:**
$\frac{4\pi}{3\sqrt{3}}$

342) Find the volume of the largest right cone that fits in a sphere of radius $1$.

343) Find the area of the largest rectangle that fits into the triangle with sides $x=0,y=0$ and $\frac{x}{4}+\frac{y}{6}=1.$

**Answer:**
$6$

344) Find the largest volume of a cylinder that fits into a cone that has base radius $R$ and height $h$.

345) Find the dimensions of the closed cylinder volume $V=16\pi$ that has the least amount of surface area.

**Answer:**
$r=2,h=4$

346) Find the dimensions of a right cone with surface area $S=4\pi$ that has the largest volume.

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For the following exercises, consider the points on the given graphs. Use a calculator to graph the functions.

347) [T] Where is the line $y=5-2x$ closest to the origin?

**Answer:**
$(2,1)$

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348) [T] Where is the line $y=5-2x$ closest to point $(1,1)$?

349) [T] Where is the parabola $y=x^2$ closest to point $(2,0)$?

**Answer:**

$(0.8351,0.6974)$

350) [T] Where is the parabola $y=x^2$ closest to point $(0,3)$?

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351) A window is composed of a semicircle placed on top of a rectangle. If you have $20ft$ of window-framing materials for the outer frame, what is the maximum size of the window you can create (with the maximum area)? Use r to represent the radius of the semicircle.


**Answer:**

Maximum area of the window is $\frac{200}{4+\pi}$ $sq$ $ft$. Here is the area function to be maximized: $A=20r-2r^2-\frac{1}{2}\pi r^2$

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For the following exercises, set up, but do not evaluate, each optimization problem.

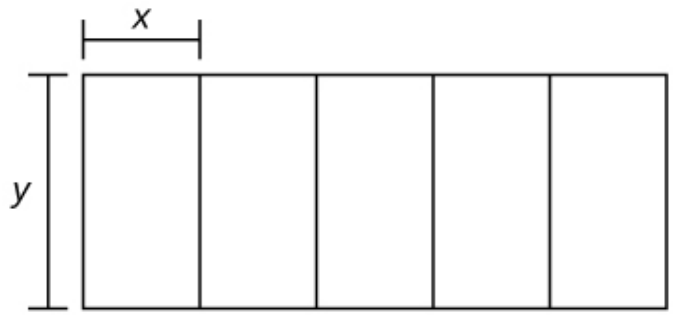
352) You have a garden row of $20$ watermelon plants that produce an average of $30$ watermelons apiece. For any additional watermelon plants planted, the output per watermelon plant drops by one watermelon. How many extra watermelon plants should you plant?

353) You are constructing a box for your cat to sleep in. The plush material for the square bottom of the box costs $\$5/ft^2$ and the material for the sides costs $\$2/ft^2$. You need a box with volume $4ft^2$. Find the dimensions of the box that minimize cost. Use $x$ to represent the length of the side of the box.

**Answer:**

$C(x)=5x^2+\frac{32}{x}$

354) You are building five identical pens adjacent to each other with a total area of $1000m^2$, as shown in the following figure. What dimensions should you use to minimize the amount of fencing?




355) You are the manager of an apartment complex with $50$ units. When you set rent at $\$800/month,$ all apartments are rented. As you increase rent by $\$25/month$, one fewer apartment is rented. Maintenance costs run $\$50/month$ for each occupied unit. What is the rent that maximizes the total amount of profit?

**Answer:**

$P(x)=(50-x)(800+25x-50)$

## Chapter Review Exercises

True or False? Justify your answer with a proof or a counterexample. Assume that $f(x)$ is continuous and differentiable unless stated otherwise.

1) If $f(-1)=-6$ and $f(1)=2$, then there exists at least one point $x \in [-1,1]$ such that $f'(x)=4.$

Solution: True, by Mean Value Theorem

2) If $f'(c)=0,$ there is a maximum or minimum at $x=c.$

3) There is a function such that $f(x)<0,f'(x)>0,$ and $f''(x)<0.$ (A graphical “proof” is acceptable for this answer.)

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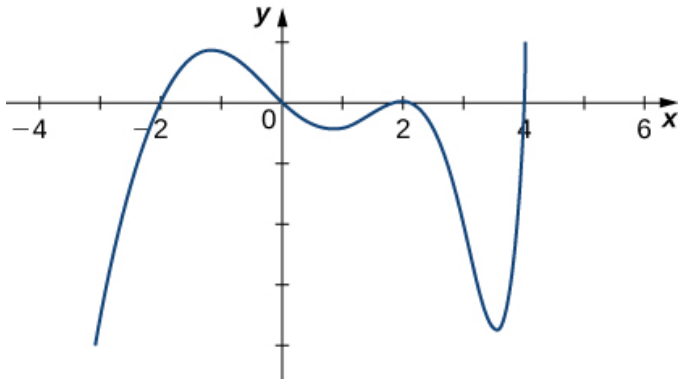
Solution: True

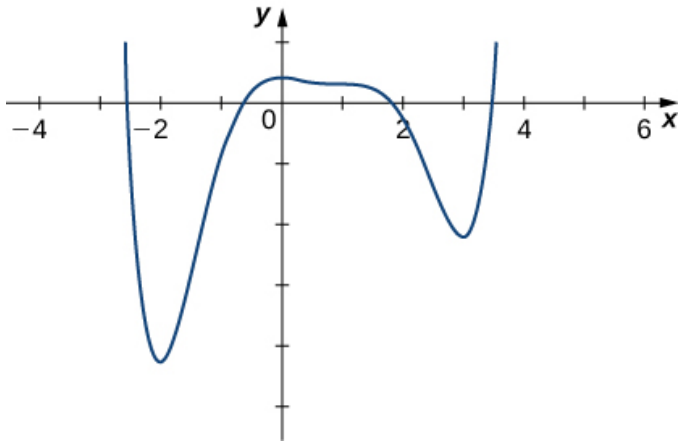
4) There is a function such that there is both an inflection point and a critical point for some value $x=a.$

5) Given the graph of $f'$, determine where $f$ is increasing or decreasing.




Solution: Increasing: $(-2,0)\cup(4,\infty)$, decreasing: $(-\infty,-2)\cup(0,4)$

6) The graph of $f$ is given below. Draw $f'$.




7) Find the linear approximation $L(x)$ to $y=x^2+\tan(\pi x)$ near $x=\frac{1}{4}.$

Solution: $L(x)=\frac{17}{16}+\frac{1}{2}(1+4\pi)(x-\frac{1}{4})$

8) Find the differential of $y=x^2-5x-6$ and evaluate for $x=2$ with $dx=0.1.$

Find the critical points and the local and absolute extrema of the following functions on the given interval.

9) $f(x)=x+\sin^2(x)$ over $[0,\pi]$

Solution: Critical point: $x=\frac{3\pi}{4},$ absolute minimum: $x=0,$ absolute maximum: $x=\pi$

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10) $f(x)=3x^4-4x^3-12x^2+6$ over $[-3,3]$

Determine over which intervals the following functions are increasing, decreasing, concave up, and concave down.

11) $x(t)=3t^4-8t^3-18t^2$

Solution: Increasing: $(-1,0)\cup(3,\infty)$, decreasing: $(-\infty,-1)\cup(0,3)$, concave up: $(-\infty,\frac{1}{3}(2-\sqrt{13}))\cup(\frac{1}{3}(2+\sqrt{13}),\infty)$, concave down: $(\frac{1}{3}(2-\sqrt{13}),\frac{1}{3}(2+\sqrt{13}))$

12) $y=x+\sin(\pi x)$

13) $g(x)=x-\sqrt{x}$

Solution: Increasing: $(\frac{1}{4},\infty)$, decreasing: $(0,\frac{1}{4})$, concave up: $(0,\infty)$, concave down: nowhere

14) $f(\theta)=\sin(3\theta)$

Evaluate the following limits.

15) $\lim_{x\to\infty}\frac{3x\sqrt{x^2+1}}{\sqrt{x4-1}}$

Solution: $3$

16) $\lim_{x\to\infty}\cos(\frac{1}{x})$

17) $\lim_{x\to 1}\frac{x-1}{\sin(\pi x)}$

Solution: $-\frac{1}{\pi}$

18) $\lim_{x\to\infty}(3x)^{1/x}$

Use Newton's method to find the first two iterations, given the starting point.

19) $y=x^3+1,x_0=0.5$

Solution: $x_1=-1,x_2=-1$

20) $\frac{1}{x+1}=\frac{1}{2},x_0=0$

Find the antiderivatives $F(x)$ of the following functions.

21) $g(x)=\sqrt{x}-\frac{1}{x^2}$

Solution: ](F(x)=\frac{2x^{3/2}}{3}+\frac{1}{x}+C$

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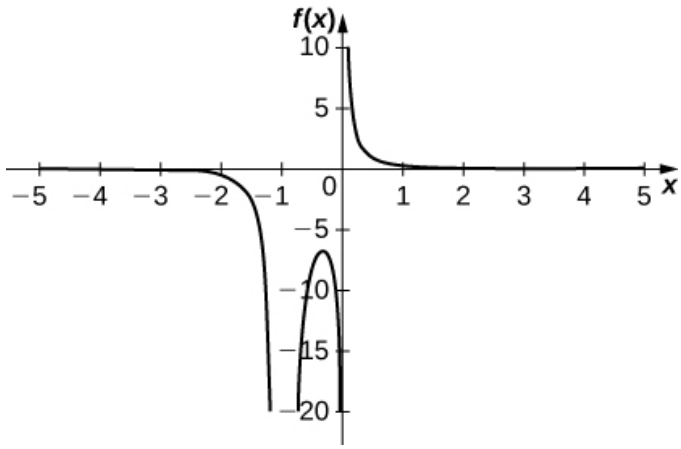
22) $f(x)=2x+6\cos x, F(\pi)=\pi^2+2$

Graph the following functions by hand. Make sure to label the inflection points, critical points, zeros, and asymptotes.

23) $y=\frac{1}{x(x+1)^2}$

Solution:




Inflection points: none; critical points: $x=-\frac{1}{3}$; zeros: none; vertical asymptotes: $x=-1, x=0$; horizontal asymptote: $y=0$

24) $y=x-\sqrt{4-x^2}$

25) A car is being compacted into a rectangular solid. The volume is decreasing at a rate of $2\ m^3/sec$. The length and width of the compactor are square, but the height is not the same length as the length and width. If the length and width walls move toward each other at a rate of $0.25$ m/sec, find the rate at which the height is changing when the length and width are $2$ m and the height is $1.5$ m.

Solution: The height is decreasing at a rate of $0.125$ m/sec

26) A rocket is launched into space; its kinetic energy is given by $K(t)=(\frac{1}{2})m(t)v(t)^2$, where $K$ is the kinetic energy in joules, $m$ is the mass of the rocket in kilograms, and $v$ is the velocity of the rocket in meters/second. Assume the velocity is increasing at a rate of $15\ m/sec^2$ and the mass is decreasing at a rate of $10$ kg/sec because the fuel is being burned. At what rate is the rocket's kinetic energy changing when the mass is $2000$ kg and the velocity is $5000$ m/sec? Give your answer in mega-Joules (MJ), which is equivalent to $10^6$ J.

27) The famous **Regiomontanus' problem** for angle maximization was proposed during the $15$ th century. A painting hangs on a wall with the bottom of the painting a distance $a$ feet above eye level, and the top $b$ feet above eye level. What distance x (in feet) from the wall should the viewer stand to maximize the angle subtended by the painting, $\theta$?

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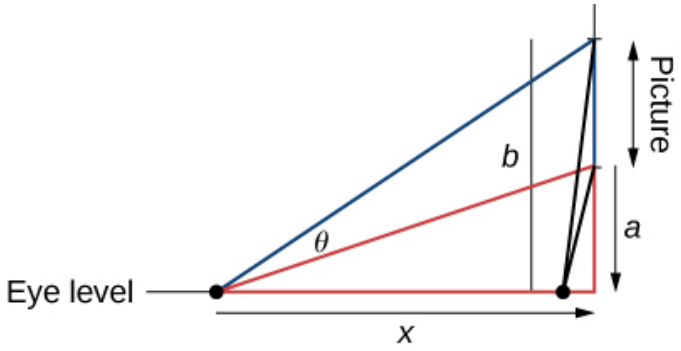


Solution: \(x=\sqrt{ab}\) feet

28) An airline sells tickets from Tokyo to Detroit for \($1200.\) There are \(500\) seats available and a typical flight books \(350\) seats. For every \($10\) decrease in price, the airline observes an additional five seats sold. What should the fare be to maximize profit? How many passengers would be onboard?

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