9.8: Rational Exponents

Learning Objectives

By the end of this section, you will be able to:

- Simplify expressions with \(a^{\frac{1}{n}}\)
- Simplify expressions with \(a^{\frac{m}{n}}\)
- Use the Laws of Exponents to simplify expressions with rational exponents

Before you get started, take this readiness quiz.

1. Add: \(\frac{7}{15} + \frac{5}{12}\).
   If you missed this problem, review [link].
2. Simplify: \((4x^2y^5)^3\).
   If you missed this problem, review [link].
3. Simplify: \((5^{\frac{1}{3}})\).
   If you missed this problem, review [link].

Simplify Expressions with \(a^{\frac{1}{n}}\)

Rational exponents are another way of writing expressions with radicals. When we use rational exponents, we can apply the properties of exponents to simplify expressions.

The Power Property for Exponents says that \(((a^m)^n = a^{m·n})\) when \(m\) and \(n\) are whole numbers. Let’s assume we are now not limited to whole numbers.
Suppose we want to find a number $p$ such that $(8^p)^3=8$. We will use the Power Property of Exponents to find the value of $p$.

\[
\begin{array}{cc} 
&{(8^p)^3=8} \\
\text{Multiply the exponents on the left.} & {8^{3p}=8} \\
\text{Write the exponent 1 on the right.} & {8^{3p}=8^1} \\
\text{The exponents must be equal.} & {3p=1} \\
\text{Solve for } p. & {p=\frac{1}{3}} \nonumber
\end{array}
\]

But we know also $((\sqrt[3]{8})^3=8)$. Then it must be that $8^{\frac{1}{3}}=\sqrt[3]{8}$.

This same logic can be used for any positive integer exponent $n$ to show that $a^{\frac{1}{n}}=\sqrt[n]{a}$.

Definition: RATIONAL EXPONENT $(a^{\frac{1}{n}})$

If $(\sqrt[n]{a})$ is a real number and $n \ge 2$, $a^{\frac{1}{n}}=\sqrt[n]{a}$.

There will be times when working with expressions will be easier if you use rational exponents and times when it will be easier if you use radicals. In the first few examples, you’ll practice converting expressions between these two notations.

Example \(\PageIndex{1}\)

Write as a radical expression:

1. $(x^{\frac{1}{2}})$
2. $(y^{\frac{1}{3}})$
3. $(z^{\frac{1}{4}})$.

Answer

We want to write each expression in the form $(\sqrt[n]{a})$.

1. \(x^{\frac{1}{2}}\)
   The denominator of the exponent is 2, so the index of the radical is 2. We do not show the index when it is 2.
   \(\sqrt{x}\)

2. \(y^{\frac{1}{3}}\)
   The denominator of the exponent is 3, so the index is 3.
   \(\sqrt[3]{y}\)

3. \(z^{\frac{1}{4}}\)
   The denominator of the exponent is 4, so the index is 4.
   \(\sqrt[4]{z}\)

Example \(\PageIndex{2}\)

Write as a radical expression:

1. $(x^{\frac{1}{2}})$
2. $(y^{\frac{1}{3}})$
3. $(z^{\frac{1}{4}})$.
1. \(t^{\frac{1}{2}}\)
2. \(m^{\frac{1}{3}}\)
3. \(r^{\frac{1}{4}}\).

**Answer**

1. \(\sqrt{t}\)
2. \(\sqrt[3]{m}\)
3. \(\sqrt[4]{r}\)

---

**Example (PageIndex{3})**

Write as a radical expression:

1. \(b^{\frac{1}{2}}\)
2. \(z^{\frac{1}{3}}\)
3. \(p^{\frac{1}{4}}\).

**Answer**

1. \(\sqrt{b}\)
2. \(\sqrt[3]{z}\)
3. \(\sqrt[4]{p}\)

---

**Example (PageIndex{4})**

Write with a rational exponent:

1. \(\sqrt{x}\)
2. \(\sqrt[3]{y}\)
3. \(\sqrt[4]{z}\).

**Answer**

We want to write each radical in the form \(a^{\frac{1}{n}}\).

1. \(\sqrt{x}\)
   
   No index is shown, so it is 2. The denominator of the exponent will be 2.

2. \(\sqrt[3]{y}\)
   
   The index is 3, so the denominator of the exponent is 3.

3. \(\sqrt[4]{z}\)
The index is 4, so the denominator of the exponent is 4. \(z^{\frac{1}{4}}\)

Example \(\PageIndex{5}\)

Write with a rational exponent:

1. \(\sqrt{s}\)
2. \(\sqrt[3]{x}\)
3. \(\sqrt[4]{b}\).

**Answer**

1. \(s^{\frac{1}{2}}\)
2. \(x^{\frac{1}{3}}\)
3. \(b^{\frac{1}{4}}\)

Example \(\PageIndex{6}\)

Write with a rational exponent:

1. \(\sqrt{v}\)
2. \(\sqrt[3]{p}\)
3. \(\sqrt[4]{p}\).

**Answer**

1. \(v^{\frac{1}{2}}\)
2. \(p^{\frac{1}{3}}\)
3. \(p^{\frac{1}{4}}\)

Example \(\PageIndex{7}\)

Write with a rational exponent:

1. \(\sqrt{5y}\)
2. \(\sqrt[3]{4x}\)
3. \(3\sqrt[4]{5z}\).

**Answer**

1. \(\sqrt{5y}\)

No index is shown, so it is 2. The denominator of the exponent will be 2. \((5y)^{\frac{1}{2}}\)

2. \(\sqrt[3]{4x}\)
The index is 3, so the denominator of the exponent is 3.

\[(4x)^{\frac{1}{3}}\]

3.

\[(3\sqrt[4]{5z})\]

The index is 4, so the denominator of the exponent is 4.

\[(3z)^{\frac{1}{4}}\]

---

Example (PageIndex{8})

Write with a rational exponent:

1. \((\sqrt{10m})\)
2. \((\sqrt[5]{3n})\)
3. \((3\sqrt[4]{6y})\).

**Answer**

1. \(((10^m)^{\frac{1}{2}})\)
2. \(((3n)^{\frac{1}{5}})\)
3. \(((486y)^{\frac{1}{4}})\)

---

Example (PageIndex{9})

Write with a rational exponent:

1. \((\sqrt[7]{3k})\)
2. \((\sqrt[4]{5j})\)
3. \((\sqrt[3]{82a})\).

**Answer**

1. \(((3k)^{\frac{1}{7}})\)
2. \(((5j)^{\frac{1}{4}})\)
3. \(((1024a)^{\frac{1}{3}})\)

---

In the next example, you may find it easier to simplify the expressions if you rewrite them as radicals first.

Example (PageIndex{10})

Simplify:

1. \((25^{\frac{1}{2}})\)
2. \((64^{\frac{1}{3}})\)
3. \((256^{\frac{1}{4}})\).

---

UC Davis ChemWiki is licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 United States License.
Answer

1. \(25^{\frac{1}{2}}\) Rewrite as a square root. \(\sqrt{25}\) Simplify. 5

2. \(64^{\frac{1}{3}}\) Rewrite as a cube root. \(\sqrt[3]{64}\) Recognize 64 is a perfect cube. \(\sqrt[3]{4^3}\) Simplify. 4

3. \(256^{\frac{1}{4}}\) Rewrite as a fourth root. \(\sqrt[4]{256}\) Recognize 256 is a perfect fourth power. \(\sqrt[4]{4^4}\) Simplify. 4

Example \(\PageIndex{11}\)

Simplify:

1. \(36^{\frac{1}{2}}\)
2. \(8^{\frac{1}{3}}\)
3. \(16^{\frac{1}{4}}\).

Answer

1. 6
2. 2
3. 2

Example \(\PageIndex{12}\)

Simplify:

1. \(100^{\frac{1}{2}}\)
2. \(27^{\frac{1}{3}}\)
3. \(81^{\frac{1}{4}}\).
Be careful of the placement of the negative signs in the next example. We will need to use the property \(a^{-n} = \frac{1}{a^n}\) in one case.

Example

Simplify:

1. \((-64)^{\frac{1}{3}}\)
2. \(-64^{\frac{1}{3}}\)
3. \((64)^{-\frac{1}{3}}\).

Answer

1. \((-64)^{\frac{1}{3}}\)
   Rewrite as a cube root.
   \(\sqrt[3]{-64}\)
   Rewrite \(-64\) as a perfect cube.
   \(\sqrt[3]{(-4)^3}\)
   Simplify.
   \(-4\)

2. \(-64^{\frac{1}{3}}\)
   The exponent applies only to the 64.
   \(-\sqrt[3]{64}\)
   Rewrite 64 as \(4^3\).
   \(-\sqrt[3]{4^3}\)
   Simplify.
   \(-4\)

3. \((64)^{-\frac{1}{3}}\)
   Rewrite as a fraction with a positive exponent, using the property, \(a^{-n} = \frac{1}{a^n}\).
   \(\frac{1}{\sqrt[3]{64}}\)
   Rewrite 64 as \(4^3\).
   \(\frac{1}{\sqrt[3]{4^3}}\)
   Simplify.
   \(\frac{1}{-4}\)

UC Davis ChemWiki is licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 United States License.
Example \(\PageIndex{14}\)

Simplify:

1. \((-125)^{\frac{1}{3}}\)
2. \(-125^{\frac{1}{3}}\)
3. \((125)^{\frac{1}{3}}\).

**Answer**

1. \(-5\)
2. \(-5\)
3. \(\frac{1}{5}\)

Example \(\PageIndex{15}\)

Simplify:

1. \((-32)^{\frac{1}{5}}\)
2. \(-32^{\frac{1}{5}}\)
3. \((32)^{\frac{1}{5}}\).

**Answer**

1. \(-2\)
2. \(-2\)
3. \(\frac{1}{2}\)

Example \(\PageIndex{16}\)

Simplify:

1. \((-16)^{\frac{1}{4}}\)
2. \(-16^{\frac{1}{4}}\)
3. \((16)^{\frac{1}{4}}\).

**Answer**

1. \((-16)^{\frac{1}{4}}\)

Rewrite as a fourth root.

\(\sqrt[4]{-16}\)

There is no real number whose fourth power is \(-16\).
2. 

The exponent applies only to the 16.

Rewrite as a fourth root.

Rewrite 16 as \(2^4\)

Simplify.

\[-2\]

3. 

Rewrite as a fraction with a positive exponent, using the property, \(a^{-n} = \frac{1}{a^n}\).

Rewrite as a fourth root.

Rewrite 16 as \(2^4\).

Simplify.

\[\frac{1}{2}\]

Example \(\PageIndex{17}\)

Simplify:

1. \((-64)^{\frac{1}{2}}\)
2. \(-64^{\frac{1}{2}}\)
3. \((64)^{-\frac{1}{2}}\).

Answer

1. \(-8\)
2. \(-8\)
3. \(\frac{1}{8}\)

Example \(\PageIndex{18}\)

Simplify:

1. \((-256)^{\frac{1}{4}}\)
2. \(-256^{\frac{1}{4}}\)
3. \((256)^{-\frac{1}{4}}\).
Simplify Expressions with \(a^{\frac{m}{n}}\)

Let’s work with the Power Property for Exponents some more.

Suppose we raise \(a^{\frac{1}{n}}\) to the power \(m\).

\[
\begin{array}{ll}
\text{Multiply the exponents.} & a^{\frac{1}{n} \cdot m} \\
\text{Simplify.} & a^{\frac{m}{n}} \\
\text{So} & a^{\frac{m}{n}} = (\sqrt[n]{a})^m \\
\end{array}
\]

Now suppose we take \(a^{m}\) to the \(\frac{1}{n}\) power.

\[
\begin{array}{ll}
\text{Multiply the exponents.} & a^{m \cdot \frac{1}{n}} \\
\text{Simplify.} & a^{\frac{m}{n}} \\
\text{So} & a^{\frac{m}{n}} = \sqrt[n]{a^m} \\
\end{array}
\]

Which form do we use to simplify an expression? We usually take the root first—that way we keep the numbers in the radicand smaller.

Definition: RATIONAL EXPONENT \(a^{\frac{m}{n}}\)

For any positive integers \(m\) and \(n\),

\[
\begin{align*}
(a^{\frac{m}{n}}) &= (\sqrt[n]{a})^m \\
(a^{\frac{m}{n}}) &= \sqrt[n]{a^m}
\end{align*}
\]

Example \(\PageIndex{19}\)

Write with a rational exponent:

1. \(\sqrt{y^3}\)
2. \(\sqrt[3]{x^2}\)
3. \(\sqrt[4]{z^3}\)

Answer

We want to use \(a^{\frac{m}{n}} = \sqrt[n]{a^m}\) to write each radical in the form \(a^{\frac{m}{n}}\).
Example \(\PageIndex{20}\)

Write with a rational exponent:

1. \(\sqrt{x^5}\)
2. \(\sqrt[4]{z^3}\)
3. \(\sqrt[5]{y^2}\).

**Answer**

1. \(x^{\frac{5}{2}}\)
2. \(z^{\frac{3}{4}}\)
3. \(y^{\frac{2}{5}}\)

Example \(\PageIndex{21}\)

Write with a rational exponent:

1. \(\sqrt[5]{a^2}\)
2. \(\sqrt[3]{b^7}\)
3. \(\sqrt[4]{m^5}\).

**Answer**

1. \(a^{\frac{2}{5}}\)
2. \(b^{\frac{7}{3}}\)
3. \(m^{\frac{5}{4}}\)

Example \(\PageIndex{22}\)

Simplify:

1. \(9^{\frac{3}{2}}\)
2. \(125^{\frac{2}{3}}\)
3. \(81^{\frac{3}{4}}\).

The numerator of the exponent is the exponent of \(y\).  
1. The denominator of the exponent is the index of the radical, \(y^\frac{1}{3}\).  
The numerator of the exponent is the exponent of \(x\).  
2. The denominator of the exponent is the index of the radical, \(x^\frac{1}{3}\).  
The numerator of the exponent is the exponent of \(z\).  
3. The denominator of the exponent is the index of the radical, \(z^\frac{1}{3}\).
We will rewrite each expression as a radical first using the property, \((a^{\frac{m}{n}}=(\sqrt[n]{a})^m)\). This form lets us take the root first and so we keep the numbers in the radicand smaller than if we used the other form.

1. \(9^{\frac{3}{2}}\)
   
   The power of the radical is the numerator of the exponent, 3. Since the denominator of the exponent is 2, this is a square root.

   Simplify.
   
   \(3^3\)
   
   27

2. \(125^{\frac{2}{3}}\)
   
   The power of the radical is the numerator of the exponent, 2. Since the denominator of the exponent is 3, this is a square root.

   Simplify.
   
   \(5^2\)
   
   25

3. \(81^{\frac{3}{4}}\)
   
   The power of the radical is the numerator of the exponent, 2. Since the denominator of the exponent is 3, this is a square root.

   Simplify.
   
   \(3^3\)
   
   27

Example \(\PageIndex{23}\)

Simplify:

1. \(4^{\frac{3}{2}}\)
2. \(27^{\frac{2}{3}}\)
3. \(625^{\frac{3}{4}}\).

Answer

1. 8
2. 9
3. 125

Example \(\PageIndex{24}\)
Simplify:

1. \(8^{\frac{5}{3}}\)
2. \(81^{\frac{3}{2}}\)
3. \(16^{\frac{3}{4}}\).

Answer

1. 32
2. 729
3. 8

Remember that \(b^{-p} = \frac{1}{b^p}\). The negative sign in the exponent does not change the sign of the expression.

Example

Simplify:

1. \(16^{\frac{-3}{2}}\)
2. \(32^{\frac{-2}{5}}\)
3. \(4^{\frac{-5}{2}}\)

Answer

We will rewrite each expression first using \(b^{-p} = \frac{1}{b^p}\) and then change to radical form.

1. \(16^{\frac{-3}{2}}\)
   
   Rewrite using \(b^{-p} = \frac{1}{b^p}\).
   
   Change to radical form. The power of the radical is the numerator of the exponent, 3. The index is the denominator of the exponent, 2.
   
   Simplify.

2. \(32^{\frac{-2}{5}}\)
   
   Rewrite using \(b^{-p} = \frac{1}{b^p}\).
   
   Change to radical form.
   
   Rewrite the radicand as a power.
   
   Simplify.
3. Rewrite using $b^{-p} = \frac{1}{b^p}$.

Change to radical form.

Simplify.

\[
\frac{1}{4^{\frac{5}{2}}}
\]

\[
\frac{1}{(\sqrt{4})^5}
\]

\[
\frac{1}{2^5}
\]

\[
\frac{1}{32}
\]

Example \(\PageIndex{26}\)

Simplify:

1. \(8^{\frac{-5}{3}}\)
2. \(81^{\frac{-3}{2}}\)
3. \(16^{\frac{-3}{4}}\).

Answer

1. \(\frac{1}{32}\)
2. \(\frac{1}{729}\)
3. \(\frac{1}{8}\)

Example \(\PageIndex{27}\)

Simplify:

1. \(4^{\frac{-3}{2}}\)
2. \(27^{\frac{-2}{3}}\)
3. \(625^{\frac{-3}{4}}\).

Answer

1. \(\frac{1}{8}\)
2. \(\frac{1}{9}\)
3. \(\frac{1}{125}\)

Example \(\PageIndex{28}\)

Simplify:

1. \((-25)^{\frac{3}{2}}\)
2. \((-25)^{\frac{-3}{2}}\)

UC Davis ChemWiki is licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 United States License.
3. \((-25)^{\frac{3}{2}}\).

Answer

1. \((-25^{\frac{3}{2}})\)
   Rewrite in radical form. \((-\sqrt{25})^3\)
   Simplify the radical \(-5^3\)
   Simplify. \(-125\)
2. \((-25^{−\frac{3}{2}})\)
   Rewrite using \(b^{−p} = \frac{1}{b^p}\). \((-\frac{1}{25^{\frac{3}{2}}})\)
   Rewrite in radical form. \((-\frac{1}{(\sqrt{25})^3})\)
   Simplify the radical. \((-\frac{1}{5^3})\)
   Simplify. \(-\frac{1}{125}\)
3. \((-25)^{\frac{3}{2}}\)
   Rewrite in radical form. \((\sqrt{-25})^3\)
   There is no real number whose square root is \(-25\). Not a real number.

Example \(\PageIndex{29}\))

Simplify:

1. \((-16^{\frac{3}{2}})\)
2. \((-16^{−\frac{3}{2}})\)
3. \((-16^{−\frac{3}{2}})\).

Answer

1. \(-64\)
2. \((-\frac{1}{64})\)
3. not a real number

Example \(\PageIndex{30}\))

Simplify:

1. \((-81^{\frac{3}{2}})\)
2. \((-81^{−\frac{3}{2}})\)
3. \((-81)^{-\frac{3}{2}}\).

**Answer**

1. \(-729\)
2. \((-\frac{1}{729})\)
3. not a real number

---

**Use the Laws of Exponents to Simplify Expressions with Rational Exponents**

The same laws of exponents that we already used apply to rational exponents, too. We will list the Exponent Properties here to have them for reference as we simplify expressions.

**SUMMARY OF EXPONENT PROPERTIES**

If \(a, b\) are real numbers and \(m, n\) are rational numbers, then

\[
\begin{array}{ll}
\textbf{Product Property} & {a^m \cdot a^n = a^{m+n}} \\
\textbf{Power Property} & {(a^m)^n = a^{m \cdot n}} \\
\textbf{Product to a Power} & {(ab)^m = a^m b^m} \\
\textbf{Quotient Property} & \frac{a^m}{a^n} = a^{m-n} , a \ne 0, m>n \\
& \frac{a^m}{a^n} = \frac{1}{a^{n-m}}, a \ne 0, n>m \\
\textbf{Zero Exponent Definition} & {a^0 = 1, a \ne 0} \\
\textbf{Quotient to a Power Property} & \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \ne 0
\end{array}
\]

When we multiply the same base, we add the exponents.

Example \(\PageIndex{31}\)

Simplify:

1. \(2^{\frac{1}{2}} \cdot 2^{\frac{5}{2}}\)
2. \(x^{\frac{2}{3}} \cdot x^{\frac{4}{3}}\)
3. \(z^{\frac{3}{4}} \cdot z^{\frac{5}{4}}\).

**Answer**

1. \(2^{\frac{1}{2} + \frac{5}{2}}\)

Add the fractions.

\(2^{\frac{6}{2}}\)

Simplify the exponent.

\(2^3\)

4

2. \((x \cdot x^{\frac{2}{3}}) \cdot x^{\frac{4}{3}}\)

The bases are the same, so we add the exponents.

\((x^{\frac{2}{3}} + \frac{4}{3})\)
The bases are the same, so we add the exponents. 

\(x^{\frac{2}{3}+\frac{4}{3}}\)

Add the fractions. 

\(x^{\frac{6}{3}}\)

Simplify. 

\(x^2\)

3. 

\(z^{\frac{3}{4}}·z^{\frac{5}{4}}\)

The bases are the same, so we add the exponents. 

\(z^{\frac{3}{4}+\frac{5}{4}}\)

Add the fractions. 

\(z^{\frac{8}{4}}\)

Simplify. 

\(z^2\)

---

Example \(\PageIndex{32}\)

Simplify:

1. \(3^{\frac{2}{3}}·3^{\frac{4}{3}}\)
2. \(y^{\frac{1}{3}}·y^{\frac{8}{3}}\)
3. \(m^{\frac{1}{4}}·m^{\frac{3}{4}}\).

**Answer**

1. 9
2. \(y^3\)
3. \(m\)

---

Example \(\PageIndex{33}\)

Simplify:

1. \(5^{\frac{3}{5}}·5^{\frac{7}{5}}\)
2. \(z^{\frac{1}{8}}·z^{\frac{7}{8}}\)
3. \(n^{\frac{2}{7}}·n^{\frac{5}{7}}\).

**Answer**

1. 25
2. \(z\)
3. \(n\)

---

We will use the Power Property in the next example.

Example \(\PageIndex{34}\)
Simplify:

1. \((x^4)^\left(\frac{1}{2}\right)\)
2. \((y^6)^\left(\frac{1}{3}\right)\)
3. \((z^9)^\left(\frac{2}{3}\right)\).

**Answer**

1. \((x^4)^\left(\frac{1}{2}\right)\)
   
   To raise a power to a power, we multiply the exponents.
   
   \(x^{4\cdot\frac{1}{2}}\)
   
   Simplify.
   
   \(x^2\)

2. \((y^6)^\left(\frac{1}{3}\right)\)

   To raise a power to a power, we multiply the exponents.
   
   \(y^{6\cdot\frac{1}{3}}\)
   
   Simplify.
   
   \(y^2\)

3. \((z^9)^\left(\frac{2}{3}\right)\)

   To raise a power to a power, we multiply the exponents.
   
   \(z^{9\cdot\frac{2}{3}}\)
   
   Simplify.
   
   \(z^6\)

Example \(\PageIndex{35}\)\)

Simplify:

1. \((p^{10})^\left(\frac{1}{5}\right)\)
2. \((q^8)^\left(\frac{3}{4}\right)\)
3. \((x^6)^\left(\frac{4}{3}\right)\)

**Answer**

1. \((p^{10})^\left(\frac{1}{5}\right)\)
   
   \((p^{10\cdot\frac{1}{5}})\)
   
   \(p^{2}\)

2. \((q^8)^\left(\frac{3}{4}\right)\)
   
   \((q^{8\cdot\frac{3}{4}})\)
   
   \(q^6\)

3. \((x^6)^\left(\frac{4}{3}\right)\)
   
   \((x^{6\cdot\frac{4}{3}})\)
   
   \(x^8\)

Example \(\PageIndex{36}\)\)

Simplify:

1. \((r^6)^\left(\frac{5}{3}\right)\)
2. \((s^{12})^\left(\frac{3}{4}\right)\)
3. \((m^9)^\left(\frac{2}{9}\right)\)

**Answer**

1. \((r^6)^\left(\frac{5}{3}\right)\)
   
   \((r^{6\cdot\frac{5}{3}})\)
   
   \(r^{10}\)

2. \((s^{12})^\left(\frac{3}{4}\right)\)
   
   \((s^{12\cdot\frac{3}{4}})\)
   
   \(s^9\)

3. \((m^9)^\left(\frac{2}{9}\right)\)
   
   \((m^{9\cdot\frac{2}{9}})\)
   
   \(m^2\)
The Quotient Property tells us that when we divide with the same base, we subtract the exponents.

Example \(\PageIndex{37}\)

Simplify:

1. \(\frac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}}\)
2. \(\frac{y^{\frac{3}{4}}}{y^{\frac{1}{4}}}\)
3. \(\frac{z^{\frac{2}{3}}}{z^{\frac{5}{3}}}\).

Answer

1. \(x^{\frac{4}{3}−\frac{1}{3}}\)
2. \(y^{\frac{3}{4}−\frac{1}{4}}\)
3. \(z^{\frac{2}{3}−\frac{5}{3}}\).

Example \(\PageIndex{38}\)

Simplify:

1. \(\frac{u^{\frac{5}{4}}}{u^{\frac{1}{4}}}\)
2. \(\frac{v^{\frac{3}{5}}}{v^{\frac{2}{5}}}\)
3. \(\frac{x^{\frac{2}{3}}}{x^{\frac{5}{3}}}\).

Answer

1. \(u\)
2. \(v^{\frac{1}{5}}\)
3. \(x^{\frac{2}{3}−\frac{5}{3}}\).

UC Davis ChemWiki is licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 United States License.
3. $\frac{1}{x}$

Example \(\PageIndex{39}\)

Simplify:

1. $\frac{c^{\frac{12}{5}}}{c^{\frac{2}{5}}}$
2. $\frac{m^{\frac{5}{4}}}{m^{\frac{9}{4}}}$
3. $\frac{d^{\frac{1}{5}}}{d^{\frac{6}{5}}}$.

Answer

1. $c^2$
2. $\frac{1}{m}$
3. $\frac{1}{d}$

Sometimes we need to use more than one property. In the next two examples, we will use both the Product to a Power Property and then the Power Property.

Example \(\PageIndex{40}\)

Simplify:

1. $(27u^{\frac{1}{2}})^{\frac{2}{3}}$
2. $(8v^{\frac{1}{4}})^{\frac{2}{3}}$.

Answer

1. $(27u^{\frac{1}{2}})^{\frac{2}{3}}$
   First we use the Product to a Power Property.
   
   $(27)^{\frac{2}{3}}(u^{\frac{1}{2}})^{\frac{2}{3}}$
   
   Rewrite 27 as a power of 3.
   
   $(3^3)^{\frac{2}{3}}(u^{\frac{1}{2}})^{\frac{2}{3}}$
   
   To raise a power to a power, we multiply the exponents.
   
   $(3^2)(u^{\frac{1}{3}})$
   
   Simplify.
   
   $9u^{\frac{1}{3}}$

2. $(8v^{\frac{1}{4}})^{\frac{2}{3}}$
   First we use the Product to a Power Property.
   
   $(8)^{\frac{2}{3}}(v^{\frac{1}{4}})^{\frac{2}{3}}$
   
   Rewrite 8 as a power of 2.
   
   $(2^3)^{\frac{2}{3}}(v^{\frac{1}{4}})^{\frac{2}{3}}$
   
   To raise a power to a power, we multiply the exponents.
   
   $(2^2)(v^{\frac{1}{6}})$
   
   Simplify.
   
   $4v^{\frac{1}{6}}$
Example \(\PageIndex{41}\)

Simplify:

1. \((32x^{\frac{1}{3}})^{\frac{3}{5}}\)
2. \((64y^{\frac{2}{3}})^{\frac{1}{3}}\).

Answer

1. \(8x^{\frac{1}{5}}\)
2. \(4y^{\frac{2}{9}}\)

Example \(\PageIndex{42}\)

Simplify:

1. \((16m^{\frac{1}{3}})^{\frac{3}{2}}\)
2. \((81n^{\frac{2}{5}})^{\frac{3}{2}}\).

Answer

1. \(64m^{\frac{1}{2}}\)
2. \(729n^{\frac{3}{5}}\)

Example \(\PageIndex{43}\)

Simplify:

1. \((m^{3}n^{9})^{\frac{1}{3}}\)
2. \((p^{4}q^{8})^{\frac{1}{4}}\).

Answer

1. \((m^{3}n^{9})^{\frac{1}{3}}\)
   
   First we use the Product to a Power Property.
   
   \((m^{3})^{\frac{1}{3}}(n^{9})^{\frac{1}{3}}\)
   
   To raise a power to a power, we multiply the exponents.
   
   \(mn^3\)

2. \((p^{4}q^{8})^{\frac{1}{4}}\)
   
   First we use the Product to a Power Property.
   
   \((p^{4})^{\frac{1}{4}}(q^{8})^{\frac{1}{4}}\)
   
   To raise a power to a power, we multiply the exponents.
   
   \(pq^2\)

We will use both the Product and Quotient Properties in the next example.

Exercise \(\PageIndex{44}\)
Simplify:

1. \(\frac{x^{\frac{3}{4}} \cdot x^{-\frac{1}{4}}}{x^{-\frac{6}{4}}}\)
2. \(\frac{y^{\frac{4}{3}} \cdot y}{y^{-\frac{2}{3}}}\).

**Answer**

1. \(\frac{x^{\frac{3}{4}} \cdot x^{-\frac{1}{4}}}{x^{-\frac{6}{4}}}\)
   
   Use the Product Property in the numerator, add the exponents.

   \(\frac{x^{-\frac{1}{4}} \cdot x^{\frac{3}{4}}}{x^{-\frac{6}{4}}}\)

   Use the Quotient Property, subtract the exponents.

   \(x^{\frac{8}{4}}\)

   Simplify.

   \(x^2\)

2. \(\frac{y^{\frac{4}{3}} \cdot y}{y^{-\frac{2}{3}}}\)

   Use the Product Property in the numerator, add the exponents.

   \(\frac{y^\frac{7}{3}}{y^{-\frac{2}{3}}}\)

   Use the Quotient Property, subtract the exponents.

   \(y^{\frac{9}{3}}\)

   Simplify.

   \(y^3\)

Example \(\PageIndex{45}\)

Simplify:

1. \(\frac{m^{\frac{2}{3}} \cdot m^{-\frac{1}{3}}}{m^{-\frac{5}{3}}}\)
2. \(\frac{n^{\frac{1}{6}} \cdot n}{n^{-\frac{11}{6}}}\).

**Answer**

1. \(m^2\)

2. \(n^3\)

Example \(\PageIndex{46}\)

Simplify:

1. \(\frac{u^{\frac{4}{5}} \cdot u^{-\frac{2}{5}}}{u^{-\frac{13}{5}}}\)
2. \(\frac{v^{\frac{1}{2}} \cdot v}{v^{-\frac{7}{2}}}\).

**Answer**

1. \(u^3\)

2. \(v^5\)
Key Concepts

- **Summary of Exponent Properties**
  - If $a,b$ are real numbers and $m,n$ are rational numbers, then
    - **Product Property** $\frac{a^m \cdot a^n}{a^{m+n}}$  
    - **Power Property** $(a^m)^n = a^{m \cdot n}$  
    - **Product to a Power** $(ab)^m = a^m b^m$  
    - **Quotient Property**:
      \[
      \frac{a^m}{a^n} = a^{m-n}, \quad a \ne 0, \quad m > n
      \]
      \[
      \frac{a^m}{a^n} = \frac{1}{a^{n-m}}, \quad a \ne 0, \quad n > m
      \]
    - **Zero Exponent Definition** $a^0 = 1, \quad a \ne 0$  
    - **Quotient to a Power Property** $(\frac{a}{b})^m = \frac{a^m}{b^m}, \quad b \ne 0$

---

Glossary

**rational exponents**

- If $\sqrt[n]{a}$ is a real number and \(n \geq 2\), $a^{\frac{1}{n}} = \sqrt[n]{a}$
- For any positive integers $m$ and $n$, $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m}$