7.4: Products and Quotients of Rational Functions

In this section we deal with products and quotients of rational expressions. Before we begin, we’ll need to establish some fundamental definitions and technique. We begin with the definition of the product of two rational numbers.

Definition

Let \( \frac{a}{b} \) and \( \frac{c}{d} \) be rational numbers. The product of these rational numbers is defined by

\[
\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}, \quad \text{or more compactly,} \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}
\]

The definition simply states that you should multiply the numerators of each rational number to obtain the numerator of the product, and you also multiply the denominators of each rational number to obtain the denominator of the product. For example,

\[
\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}
\]

Of course, you should also check to make sure your final answer is reduced to lowest terms.

Let’s look at an example.

Example \( \PageIndex{1} \)

Simplify the product of rational numbers \( \frac{6}{231} \cdot \frac{35}{10} \)

Solution
First, multiply numerators and denominators together as follows.

\[
\frac{6}{231} \cdot \frac{35}{10} = \frac{6 \cdot 35}{231 \cdot 10} = \frac{210}{2310}
\]

However, the answer is not reduced to lowest terms. We can express the numerator as a product of primes.

\[
210 = 21 \cdot 10 = 3 \cdot 7 \cdot 2 \cdot 5 = 2 \cdot 3 \cdot 5 \cdot 7
\]

It’s not necessary to arrange the factors in ascending order, but every little bit helps. The denominator can also be expressed as a product of primes.

\[
2310 = 10 \cdot 231 = 2 \cdot 5 \cdot 7 \cdot 33 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11
\]

We can now cancel common factors.

\[
\frac{210}{2310} = \frac{2 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 3 \cdot 5 \cdot 7 \cdot 11} = \frac{\not{2} \cdot \not{3} \cdot \not{5} \cdot \not{7}}{\not{2} \cdot \not{3} \cdot \not{5} \cdot \not{7} \cdot 11} = \frac{1}{11}
\]

However, this approach is not the most efficient way to proceed, as multiplying numerators and denominators allows the products to grow to larger numbers, as in 210/2310. It is then a little bit harder to prime factor the larger numbers.

A better approach is to factor the smaller numerators and denominators immediately, as follows.

\[
\frac{6}{231} \cdot \frac{35}{10} = \frac{2 \cdot 3}{3 \cdot 7 \cdot 11} \cdot \frac{5 \cdot 7}{2 \cdot 5}
\]

We could now multiply numerators and denominators, then cancel common factors, which would match identically the last computation in equation (5).

However, we can also employ the following cancellation rule.

Cancellation Rule

When working with the product of two or more rational expressions, factor all numerators and denominators, then cancel. The cancellation rule is simple: cancel a factor “on the top” for an identical factor “on the bottom.” Speaking more technically, cancel any factor in any numerator for an identical factor in any denominator.

Thus, we can finish our computation by canceling common factors, canceling “something on the top for something on the bottom.”

\[
\frac{6}{231} \cdot \frac{35}{10} = \frac{2 \cdot 3 \cdot 7 \cdot 11 \cdot \frac{\not{5}}{\not{2} \cdot \not{3} \cdot \not{7} \cdot 11} \cdot \frac{\not{5} \cdot \not{7}}{\not{2} \cdot \not{5}} = \frac{1}{11}
\]

Note that we canceled a 2, 3, 5, and a 7 “on the top” for a 2, 3, 5, and 7 “on the bottom.”

Thus, we have two choices when multiplying rational expressions:
• Multiply numerators and denominators, factor, then cancel.
• Factor numerators and denominators, cancel, then multiply numerators and denominators.

It is the latter approach that we will use in this section. Let’s look at another example.

Example \(\PageIndex{2}\)

Simplify the expression \[\frac{x^2-x-6}{x^2+2x-15} \cdot \frac{x^2-x-30}{x^2-2x-8}\] State restrictions.

Solution

Use the ac-test to factor each numerator and denominator. Then cancel as shown.

\[
\begin{aligned}
\frac{x^2-x-6}{x^2+2x-15} \cdot \frac{x^2-x-30}{x^2-2x-8} &= \frac{(x+2)(x-3)}{(x-3)(x+5)} \\
\cdot \frac{(x+5)(x-6)}{(x+2)(x-4)} &= \frac{(x+2)(x-3)}{(x-3)(x+5)} \cdot \frac{(x+5)(x-6)}{(x+2)(x-4)} \\\n&= \frac{x-6}{x-4}
\end{aligned}
\]

The first fraction’s denominator has factors \(x - 3\) and \(x + 5\). Hence, \(x = 3\) or \(x = -5\) will make this denominator zero. Therefore, the 3 and -5 are restrictions.

The second fraction’s denominator has factors \(x + 2\) and \(x - 4\). Hence, \(x = -2\) or \(x = 4\) will make this denominator zero. Therefore, -2 and 4 are restrictions. For all values of \(x\), except the restrictions -5, -2, 3, and 4, the left side of
\[
\frac{x^2-x-6}{x^2+2x-15} \cdot \frac{x^2-x-30}{x^2-2x-8} = \frac{x-6}{x-4}
\]
is identical to its right side.

It’s possible to use your graphing calculator to check your results. First, load the left- and right-hand sides of equation (8) into the calculator’s into Y1 and Y2 in your graphing calculator’s Y= menu, as shown in Figure \(\PageIndex{1}\)(a). Press 2nd TBLSET and set TblStart = -6 and \(\Delta \text{Tbl} \ 1\), as shown in Figure \(\PageIndex{1}\)(b). Make sure that AUTO is highlighted and selected with the ENTER key on both the independent and dependent variables. Press 2nd TABLE to produce the tabular display in Figure \(\PageIndex{1}\)(c).

Figure \(\PageIndex{1}\). Using the table features of the graphing calculator to check the result in equation (8).

Remember that the left- and right-hand sides of equation (8) are loaded in Y1 and Y2, respectively.

• In Figure \(\PageIndex{1}\)(c), note the ERR (error) message at the restricted values of \(x = -5\) and \(x = -2\). However, other than at these two restrictions, the functions Y1 and Y2 agree at all other values of \(x\) in Figure \(\PageIndex{1}\)(c).

• Use the down arrow to scroll down in the table to produce the tabular results shown in Figure \(\PageIndex{1}\)(d). Note the ERR (error) message at the restricted values of \(x = 3\) and \(x = 4\). However, other than at these two restrictions, the functions Y1 and Y2 agree at all other values of \(x\) in Figure \(\PageIndex{1}\)(d).
• If you scroll up or down in the table, you’ll find that the functions Y1 and Y2 agree at all values of x other than the restricted values −5, −2, 3, and 4.

Let’s look at another example.

Example \(\PageIndex{3}\)

Simplify \[
\frac{9-x^{2}}{x^{2}+3 x} \cdot \frac{6 x-2 x^{2}}{x^{2}-6 x+9}\]
State any restrictions.

Solution

The first numerator can be factored using the difference of two squares pattern.

\[9-x^{2}=(3+x)(3-x)\]

The second denominator is a perfect square trinomial and can be factored as the square of a binomial.

\[x^{2}-6 x+9=(x-3)^{2}\]

You will want to remove the greatest common factor from the first denominator and second numerator.

\[x^{2}+3 x=x(x+3) \quad \text{and} \quad 6 x-2 x^{2}=2 x(3-x)\]

Thus, \[
\frac{9-x^{2}}{x^{2}+3 x} \cdot \frac{6 x-2 x^{2}}{x^{2}-6 x+9} = \frac{(3+x)(3-x)}{x(x+3)} \cdot \frac{-2 x(x-3)}{(x-3)^{2}}
\]

We’ll need to execute a sign change or two to create common factors in the numerators and denominators. So, in both the first and second numerator, factor a −1 from the factor 3 − x to obtain 3 − x = −1(x − 3). Because the order of factors in a product doesn’t matter, we’ll slide the −1 to the front in each case.

\[
\frac{9-x^{2}}{x^{2}+3 x} \cdot \frac{6 x-2 x^{2}}{x^{2}-6 x+9} = \frac{-(3+x)(x-3)}{x(x+3)} \cdot \frac{-2 x(x-3)}{(x-3)^{2}}
\]

We can now cancel common factors.

\[
\begin{aligned}
\frac{9-x^{2}}{x^{2}+3 x} \cdot \frac{6 x-2 x^{2}}{x^{2}-6 x+9} &= \frac{-(3+x)(x-3)}{x(x+3)} \cdot \frac{-2 x(x-3)}{(x-3)^{2}} \\
&= 2
\end{aligned}
\]

A few things to notice:

• The factors 3 + x and x + 3 are identical, so they may be cancelled, one on the top for one on the bottom.
• Two factors of x − 3 on the top are cancelled for (x − 3)(x + 3) (which is equivalent to (x − 3)(x − 3)) on the bottom.
• An x on top cancels an x on the bottom.
• We’re left with two minus signs (two −1’s) and a 2. So the solution is a positive 2.

Finally, the first denominator has factors x and x + 3, so x = 0 and x = −3 are restrictions (they make this denominator equal to

UC Davis ChemWiki is licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 United States License.
zero). The second denominator has two factors of $x - 3$, so $x = 3$ is an additional restriction.

Hence, for all values of $x$, except the restricted values $-3, 0,$ and $3$, the left-hand side of \[rac{9-x^2}{x^2+3x} \cdot \frac{6x-2x^2}{x^2-6x+9}=2\]
is identical to the right-hand side. Again, this claim is easily tested on the graphing calculator which is evidenced in the sequence of screen captures in Figure (PageIndex{2}).

An alternate approach to the problem in equation (10) is to note differing orders in the numerators and denominators (descending, ascending powers of $x$) and anticipate the need for a sign change. That is, make the sign change before you factor.

For example, negate (multiply by $-1$) both numerator and fraction bar of the first fraction to obtain

\[
\frac{9-x^2}{x^2+3x} = -\frac{x^2-9}{x^2+3x}
\]

According to our sign change rule, negating any two parts of a fraction leaves the fraction unchanged.

If we perform a similar sign change on the second fraction (negate numerator and fraction bar), then we can factor and cancel common factors.

\[
\begin{aligned}
\frac{9-x^2}{x^2+3x} \cdot \frac{6x-2x^2}{x^2-6x+9} &=-\frac{x^2-9}{x^2+3x} \cdot-\frac{2x^2-6x}{x^2-6x+9} \\
&=-\frac{(x+3)(x-3)}{x(x+3)} \cdot-\frac{2x(x-3)^2}{(x-3)^2} \\
&=2
\end{aligned}
\]

**Division of Rational Expressions**

A simple definition will change a problem involving division of two rational expressions into one involving multiplication of two rational expressions. Then there’s nothing left to explain, for we already know how to multiply two rational expressions.

So, let’s motivate our definition of division. Suppose we ask the question, how many halves are in a whole? The answer is easy, as two halves make a whole. Thus, when we divide 1 by 1/2, we should get 2. There are two halves in one whole.

Let’s raise the stakes a bit and ask how many halves are in six? To make the problem more precise, imagine you’ve ordered 6 pizzas and you cut each in half. How many halves do you have? Again, this is easy when you think about the problem in this manner, the answer is 12. Thus,
(how many halves are in six) is identical to

$\frac{6}{2}$

which, of course, is 12. Hopefully, thanks to this opening motivation, the following definition will not seem too strange.

**Definition**

To perform the division $\frac{a}{b} \div \frac{c}{d}$, invert the second fraction and multiply, as in $\frac{a}{b} \cdot \frac{d}{c}$

Thus, if we want to know how many halves are in 12, we change the division into multiplication (“invert and multiply”).

$12 \div \frac{1}{2} = 12 \cdot 2 = 24$

This makes sense, as there are 24 “halves” in 12. Let’s look at a harder example.

**Example**

Simplify $\frac{33}{15} \div \frac{14}{10}$

**Solution**

Invert the second fraction and multiply. After that, all we need to do is factor numerators and denominators, then cancel common factors.

$\frac{33}{15} \div \frac{14}{10} = \frac{33}{15} \cdot \frac{10}{14} = \frac{3 \cdot 11}{3 \cdot 5} \cdot \frac{2 \cdot 5}{2 \cdot 7} = \frac{11}{7}$

An interesting way to check this result on your calculator is shown in the sequence of screens in Figure 3.

**Figure 3**. Using the calculator to check division of fractions.

After entering the original problem in your calculator, press ENTER, then press the MATH button, then select 1:I Frac from the menu and press ENTER. The result is shown in Figure (3)(c), which agrees with our calculation above.

Let’s look at another example.

**Example**

Simplify $\frac{9 + 3 \cdot x - 2 \cdot x^2}{x^2 - 16} \div \frac{4 \cdot x^3 - 9 \cdot x}{2 \cdot x^2 + 5 \cdot x - 12}$ State the restrictions.
**Solution**

Note the order of the first numerator differs from the other numerators and denominators, so we “anticipate” the need for a sign change, negating the numerator and fraction bar of the first fraction. We also invert the second fraction and change the division to multiplication (“invert and multiply”).

\[-\frac{2 x^{2}-3 x-9}{x^{2}-16} \cdot \frac{2 x^{2}+5 x-12}{4 x^{3}-9 x}\]

The numerator in the first fraction in equation (17) is a quadratic trinomial, with ac = (2)(−9) = −18. The integer pair 3 and −6 has product −18 and sum −3. Hence,

\[
\begin{aligned}
2 x^{2}-3 x-9 &=2 x^{2}+3 x-6 x-9 \\
&=x(2 x+3)-3(2 x+3) \\
&=(x-3)(2 x+3)
\end{aligned}
\]

The denominator of the first fraction in equation (17) easily factors using the difference of two squares pattern.

\[
x^{2}-16=(x+4)(x-4)
\]

The numerator of the second fraction in equation (17) is a quadratic trinomial, with ac = (2)(−12) = −24. The integer pair −3 and 8 have product −24 and sum 5. Hence,

\[
\begin{aligned}
2 x^{2}+5 x-12 &=2 x^{2}-3 x+8 x-12 \\
&=x(2 x-3)+4(2 x-3) \\
&=(x+4)(2 x-3)
\end{aligned}
\]

To factor the denominator of the last fraction in equation (17), first pull the greatest common factor (in this case x), then complete the factorization using the difference of two squares pattern.

\[
4 x^{3}-9 x=x(4 x^{2}-9)=x(2 x+3)(2 x-3)
\]

We can now replace each numerator and denominator in equation (17) with its factorization, then cancel common factors.

\[
\begin{aligned}
-\frac{(x-3)(2 x+3)}{(x+4)(x-4)} &\cdot \frac{(x+4)(2 x-3)}{x(2 x+3)(2 x-3)} \\
&=\frac{-x-3}{x(x-4)}
\end{aligned}
\]

The last denominator has factors x and x − 4, so x = 0 and x = 4 are restrictions. In the body of our work, the first fraction’s denominator has factors x + 4 and x − 4. We’ve seen the factor x − 4 already, so only the factor x + 4 adds a new restriction, x = −4. Again, in the body of our work, the second fraction’s denominator has factors x, 2x + 3, and 2x − 3, so we have added restrictions x = 0, x = −3/2, and x = 3/2.

There’s one bit of trickery here that can easily be overlooked. In the body of our work, the second fraction’s numerator was originally a denominator before we inverted the fraction. So, we must consider what makes this numerator zero as well. Fortunately, the factors in this numerator are x + 4 and 2x − 3 and we’ve already considered the restrictions produced by these factors.

Hence, for all values of x, except the restricted values −4, −3/2, 0, 3/2, and 4, the left-hand side of
\[\frac{9+3x-2x^2}{x^2-16} \div \frac{4x^3-9x}{2x^2+5x-12} = \frac{x-3}{x(x-4)}\]

is identical to the right-hand side.

Again, this claim is easily checked by using a graphing calculator, as is partially evidenced (you’ll have to scroll downward to see the last restriction come into view) in the sequence of screen captures in Figure \(\PageIndex{4}\).

![Figure \(\PageIndex{4}\). Using the table feature of the calculator to check the result in equation (18).](image)

Alternative Notation. Note that the fractional expression \(a/b\) means “a divided by b,” so we can use this equivalent notation for \(a \div b\). For example, the expression

\[\frac{9+3x-2x^2}{x^2-16} \div \frac{4x^3-9x}{2x^2+5x-12}\]

is equivalent to the expression

\[\frac{\frac{9+3x-2x^2}{x^2-16}}{\frac{4x^3-9x}{2x^2+5x-12}}\]

Let’s look at an example of this notation in use.

Example \(\PageIndex{6}\)

Given that \(f(x)=\frac{x}{x+3}\) and \(g(x)=\frac{x^2}{x+3}\), simplify both \(f(x)g(x)\) and \(f(x)/g(x)\).

**Solution**

First, the multiplication. There is no possible cancellation, so we simply multiply numerators and denominators.

\[f(x)g(x)=\frac{x}{x+3} \cdot \frac{x^2}{x+3} = \frac{x^3}{(x+3)^2}\]

This result is valid for all values of \(x\) except \(-3\).

On the other hand, \[\frac{f(x)}{g(x)}=\frac{\frac{x}{x+3}}{\frac{x^2}{x+3}} = \frac{x}{x+3} \div \frac{x^2}{x+3}\]

When we “invert and multiply,” then cancel, we obtain

\[\frac{f(x)}{g(x)}=\frac{x}{x+3} \cdot \frac{x+3}{x^2} = \frac{1}{x}\]

This result is valid for all values of \(x\) except \(-3\) and 0.

**Exercise**

In Exercises 1-10, reduce the product to a single fraction in lowest terms.

Exercise \(\PageIndex{1}\)
\( \frac{108}{14} \cdot \frac{6}{100} \)

**Answer**

\( \frac{81}{175} \)

**Exercise \PageIndex{2} \)**

\( \frac{75}{63} \cdot \frac{18}{45} \)

**Exercise \PageIndex{3} \)**

\( \frac{189}{56} \cdot \frac{12}{27} \)

**Answer**

\( \frac{3}{2} \)

**Exercise \PageIndex{4} \)**

\( \frac{45}{72} \cdot \frac{63}{64} \)

**Exercise \PageIndex{5} \)**

\( \frac{15}{36} \cdot \frac{28}{100} \)

**Answer**

\( \frac{7}{60} \)

**Exercise \PageIndex{6} \)**

\( \frac{189}{49} \cdot \frac{32}{25} \)

**EXERCISE \PageIndex{7} \)**

\( \frac{21}{100} \cdot \frac{125}{16} \)

**Answer**

\( \frac{105}{64} \)

**Exercise \PageIndex{8} \)**

\( \frac{21}{35} \cdot \frac{49}{45} \)

**Exercise \PageIndex{9} \)**
\( \frac{56}{20} \cdot \frac{98}{32} \)

**Answer**

\( \frac{343}{40} \)

Exercise \( \PageIndex{10} \)

\( \frac{27}{125} \cdot \frac{4}{12} \)

In **Exercises 11-34**, multiply and simplify. State all restrictions.

Exercise \( \PageIndex{11} \)

\( \frac{x+6}{x^2+16x+63} \cdot \frac{x^2+7x}{x+4} \)

**Answer**

Provided \( x \neq -9, -7, -4 \),

\( \frac{x(x+6)}{(x+9)(x+4)} \)

Exercise \( \PageIndex{12} \)

\( \frac{x^2+9x}{x^2−25} \cdot \frac{x^2−x−20}{−18−11x−x^2} \)

Exercise \( \PageIndex{13} \)

\( \frac{x^2+7x+10}{x^2−1} \cdot \frac{−9+10x−x^2}{x^2+9x+20} \)

**Answer**

Provided \( x \neq 1, −1, −4, −5 \),

\( −\frac{(x+2)(x−9)}{(x+1)(x+4)} \)

Exercise \( \PageIndex{14} \)

\( \frac{x^2+5x}{x−4} \cdot \frac{x−2}{x^2+6x+5} \)

Exercise \( \PageIndex{15} \)

\( \frac{x^2−5x}{x^2+2x−48} \cdot \frac{x^2+11x+24}{x^2−x} \)

**Answer**

Provided \( x \neq −8, 6, 1, 0 \),

UC Davis ChemWiki is licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 United States License.
\[
\frac{(x-5)(x+3)}{(x-6)(x-1)}
\]

Exercise \(\PageIndex{16}\)

\[
\frac{x^2-6x-27}{x^2+10x+24} \cdot \frac{x^2+13x+42}{x^2-11x+18}
\]

Exercise \(\PageIndex{17}\)

\[
\frac{-x-x^2}{x^2-9x+8} \cdot \frac{x^2-4x+3}{x^2+4x+3}
\]

**Answer**

Provided \(x \neq 1, 8, -3, -1\),

\[
-\frac{x(x-3)}{(x-8)(x+3)}
\]

Exercise \(\PageIndex{18}\)

\[
\frac{x^2-12x+35}{x^2+2x-15} \cdot \frac{45+4x-x^2}{x^2+x-30}
\]

Exercise \(\PageIndex{19}\)

\[
\frac{x+2}{7-x} \cdot \frac{x^2+x-56}{x^2+7x+6}
\]

**Answer**

Provided \(x \neq 7, -1, -6\),

\[
-\frac{(x+2)(x+8)}{(x+1)(x+6)}
\]

Exercise \(\PageIndex{20}\)

\[
\frac{x^2-2x-15}{x^2+x} \cdot \frac{x^2+7x}{x^2+12x+27}
\]

Exercise \(\PageIndex{21}\)

\[
\frac{x^2-9}{x^2-4x-45} \cdot \frac{x-6}{-3-x}
\]

**Answer**

Provided \(x \neq -3, -5, 9\),

\[
-\frac{(x-3)(x-6)}{(x-5)(x-3)}
\]

Exercise \(\PageIndex{22}\)

\[
\frac{x^2-12x+27}{x-5} \cdot \frac{x-4}{x^2-18x+8}
\]
Exercise \(\PageIndex{23}\)

\[
\frac{x+5}{x^2-2x-24} \cdot \frac{x^2+12x+32}{x+7}
\]

**Answer**

Provided \(x \neq -8, -4, -7\),

\[
\frac{(x+5)(x-6)}{(x+8)(x+7)}
\]

Exercise \(\PageIndex{24}\)

\[
\frac{x^2-36}{x^2+11x+24} \cdot \frac{-8-x}{x+4}
\]

Exercise \(\PageIndex{25}\)

\[
\frac{x-5}{x^2-8x+12} \cdot \frac{x^2-12x+36}{x-8}
\]

**Answer**

Provided \(x \neq 2, 6, 8\),

\[
\frac{(x-5)(x-6)}{(x-2)(x-8)}
\]

Exercise \(\PageIndex{26}\)

\[
\frac{x^2-5x-36}{x-1} \cdot \frac{x-5}{x^2-81}
\]

Exercise \(\PageIndex{27}\)

\[
\frac{x^2+2x-15}{x^2-10x+16} \cdot \frac{x^2-7x+10}{3x^2+13x-10}
\]

**Answer**

Provided \(x \neq 2, 8, \frac{2}{3}, -5\),

\[
\frac{(x-3)(x-5)}{(3x-2)(x-8)}
\]

Exercise \(\PageIndex{28}\)

\[
\frac{5x^2+14x-3}{x+9} \cdot \frac{x-7}{x^2+10x+21}
\]

Exercise \(\PageIndex{29}\)

\[
\frac{x^2-4}{x^2+2x-63} \cdot \frac{x^2+6x-27}{x^2-6x-16}
\]
Exercise \(\PageIndex{30}\)

\[
\frac{x^2+5x+6}{x^2-3x} \cdot \frac{x^2-5x}{x^2+9x+18}
\]

Exercise \(\PageIndex{31}\)

\[
\frac{x-1}{x^2+2x-63} \cdot \frac{x^2-81}{x+4}
\]

Answer

Provided \(x \ne 7, -9, -4\),

\[
\frac{(x-1)(x-9)}{(x-7)(x+4)}
\]

Exercise \(\PageIndex{32}\)

\[
\frac{x^2+9x}{x^2+7x+12} \cdot \frac{27+6x-x^2}{x^2-5x}
\]

Exercise \(\PageIndex{33}\)

\[
\frac{5-x}{x+3} \cdot \frac{x^2+3x-18}{2x^2-7x-15}
\]

Answer

Provided \(x \ne -3, -\frac{3}{2}, 5\)

\[
-\frac{(x+6)(x-3)}{(2x+3)(x+3)}
\]

Exercise \(\PageIndex{34}\)

\[
\frac{4x^2+21x+5}{18-7x-x^2} \cdot \frac{x^2+11x+18}{x^2-25}
\]

In Exercises 35-58, divide and simplify. State all restrictions.

Exercise \(\PageIndex{35}\)

\[
\frac{\frac{x^2-14x+48}{x^2+10x+16}}{\frac{-24+11x-x^2}{x^2-2-x^2}}
\]

Answer

Provided \(x \ne -8, -2, 9, 3, 8\),
\[
-\frac{(x-6)(x-9)}{(x+2)(x-3)}
\]

Exercise \(\PageIndex{36}\)

\[
\frac{x-1}{x^2-14x+48} \div \frac{x+5}{x^2-3x-18}
\]

Exercise \(\PageIndex{37}\)

\[
\frac{x^2-1}{x^2-7x+12} \div \frac{x^2+6x+5}{-24+10x-x^2}
\]

Answer

Provided \(x \ne 4, 3, 6, -5, -1\),

\[
-\frac{(x-1)(x-6)}{(x-3)(x+5)}
\]

Exercise \(\PageIndex{38}\)

\[
\frac{x^2-13x+42}{x^2-2x-63} \div \frac{x^2-x-42}{x^2+8x+7}
\]

Exercise \(\PageIndex{39}\)

\[
\frac{x^2-25}{x+1} \div \frac{5x^2+23x-10}{x-3}
\]

Answer

Provided \(x \ne -1, \frac{2}{5}, -5, 3\),

\[
\frac{(x-5)(x-3)}{(5x-2)(x+1)}
\]

Exercise \(\PageIndex{40}\)

\[
\frac{\frac{x^2-3x}{x^2-7x+6}}{\frac{x^2-4x}{3x^2-11x-42}}
\]

Exercise \(\PageIndex{41}\)

\[
\frac{\frac{x^2+10x+21}{x-4}}{\frac{x^2+3x}{x+8}}
\]

Answer

Provided \(x \ne 4, 0, -3, -8\),

\[
\frac{(x+7)(x+8)}{x(x-4)}
\]

Exercise \(\PageIndex{42}\)

\[
\frac{x^2+8x+15}{x^2-14x+45} \div \frac{x^2+11x+30}{-30+11x-x^2}
\]
Exercise \(\PageIndex{43}\))

\[\frac{\frac{x^2-6x-16}{x^2+x-42}}{\frac{x^2-64}{x^2+12x+35}}\]

**Answer**

Provided \((x \neq -7, 6, -5, -8, 8)\),

\[\frac{(x+2)(x+5)}{(x-6)(x+8)}\]

Exercise \(\PageIndex{44}\))

\[\frac{\frac{x^2+3x+2}{x^2-9x+18}}{\frac{x^2+7x+6}{x^2-6x}}\]

Exercise \(\PageIndex{45}\))

\[\frac{\frac{x^2+12x+35}{x+4}}{\frac{x^2+10x+25}{x+9}}\]

**Answer**

Provided \((x \neq -4, -5, -9)\),

\[\frac{(x+7)(x+9)}{(x+4)(x+5)}\]

Exercise \(\PageIndex{46}\))

\[\frac{x^2-8x+7}{x^2+3x-18} \div \frac{x^2-7x}{x^2+6x-27}\]

Exercise \(\PageIndex{47}\))

\[\frac{x^2+x-30}{x^2+5x-36} \div \frac{-6-x}{x+8}\]

**Answer**

Provided \((x \neq 4, -9, -8, -6)\),

\[-\frac{(x-5)(x+8)}{(x-4)(x+9)}\]

Exercise \(\PageIndex{48}\))

\[\frac{2x-x^2}{x^2-15x+54} \div \frac{x^2+x}{x^2-11x+30}\]

Exercise \(\PageIndex{49}\))

\[\frac{x^2-9x+8}{x^2-9} \div \frac{x^2-8x-15}{-8x-x^2}\]
Answer
Provided \((x \neq -3, 3, -5, 0, 8)\),
\((-\frac{(x-1)(x+5)}{x(x-3)})\)

Exercise \(\PageIndex{50}\)
\(\frac{x+5}{x^2+2x+1} \div \frac{x-2}{x^2+10x+9}\)

Exercise \(\PageIndex{51}\)
\(\left(\frac{x^2-4}{x+8}\right) \left(\frac{x^2-10x+16}{x+3}\right)\)

Answer
Provided \((x \neq -8, 8, 2, -3)\),
\(\frac{(x+2)(x+3)}{(x+8)(x-8)}\)

Exercise \(\PageIndex{52}\)
\(\frac{27-6x-x^2}{x^2+9x+20} \div \frac{x^2-12x+27}{x^2+5x}\)

Exercise \(\PageIndex{53}\)
\(\left(\frac{x^2+5x+6}{x^2-36}\right) \left(\frac{x-7}{-6-x}\right)\)

Answer
Provided \((x \neq 6, -6, 7)\),
\((-\frac{(x+2)(x+3)}{(x-6)(x-7)})\)

Exercise \(\PageIndex{54}\)
\(\frac{2-x}{x-5} \div \frac{x^2+3x-10}{x^2-14x+48}\)

Exercise \(\PageIndex{55}\)
\(\left(\frac{x+3}{x^2+4x-12}\right) \left(\frac{x-4}{x^2-36}\right)\)

Answer
Provided \((x \neq 2, -6, 4, 6)\),
\(\frac{(x+3)(x-6)}{(x-2)(x-4)}\)
Exercise \(\PageIndex{56}\)
\[
\frac{x+3}{x^2−x−2} \div \frac{x}{x^2−3x−4}
\]

Exercise \(\PageIndex{57}\)
\[
\frac{x^2−11x+28}{x^2+5x+6} \div \frac{7x^2−30x+8}{x^2−x−6}
\]

Answer

Provided \(x \neq −2, −3, \frac{2}{7}, 4\),
\[
\frac{(x−7)(x−3)}{(7x−2)(x+3)}
\]

Exercise \(\PageIndex{58}\)
\[
\frac{\frac{x−7}{3−x}}{\frac{2x^2+3x−5}{x^2−12x+27}}
\]

Exercise \(\PageIndex{59}\)
Let
\[
f(x) = \frac{x^2−7x+10}{x^2+4x−21}
\]
and
\[
g(x) = \frac{5x−x^2}{x^2+15x+56}
\]
Compute \(\frac{f(x)}{g(x)}\) and simplify your answer.

Answer

Provided \(x \neq −7, 3, −8, 0, 5\),
\[
−\frac{(x−2)(x+8)}{x(x−3)}
\]

Exercise \(\PageIndex{60}\)
Let
\[
f(x) = \frac{x^2+15x+56}{x^2−x−20}
\]
and
\[
g(x) = \frac{−7−x}{x+1}
\]
Compute \(\frac{f(x)}{g(x)}\) and simplify your answer.
Exercise \(\PageIndex{61}\)

Let

\[ f(x) = \frac{x^2+12x+35}{x^2+4x−32} \]

and

\[ g(x) = \frac{x^2−2x−35}{x^2+8x} \]

Compute \(\frac{f(x)}{g(x)}\) and simplify your answer.

**Answer**

Provided \(x \neq −8, 4, 0, 7, −5\),

\[ \frac{x(x+7)}{(x−4)(x−7)} \]

Exercise \(\PageIndex{62}\)

Let

\[ f(x) = \frac{x^2+4x+3}{x−1} \]

and

\[ g(x) = \frac{x^2−4x−21}{x+5} \]

Compute \(\frac{f(x)}{g(x)}\) and simplify your answer.

Exercise \(\PageIndex{63}\)

Let

\[ f(x) = \frac{x^2+x−20}{x} \]

and

\[ g(x) = \frac{x−1}{x^2−2x−35} \]

Compute \(f(x)g(x)\) and simplify your answer.

**Answer**

Provided \(x \neq 0, 7, −5\),

\[ \frac{(x−4)(x−1)}{x(x−7)} \]
Exercise \(\PageIndex{64}\)

Let

\[
f(x) = \frac{x^2 + 10x + 24}{x^2 - 13x + 42}
\]

and

\[
g(x) = \frac{x^2 - 6x - 7}{x^2 + 8x + 12}
\]

Compute \(f(x)g(x)\) and simplify your answer.

Exercise \(\PageIndex{65}\)

Let

\[
f(x) = \frac{x + 5}{-6 - x}
\]

and

\[
g(x) = \frac{x^2 + 8x + 12}{x^2 - 49}
\]

Compute \(f(x)g(x)\) and simplify your answer.

**Answer**

Provided \(x \neq -6, -7, 7\),

\[
-\frac{(x+5)(x+2)}{(x+7)(x-7)}
\]

Exercise \(\PageIndex{66}\)

Let

\[
f(x) = \frac{8 - 7x - x^2}{x^2 - 8x - 9}
\]

and

\[
g(x) = \frac{x^2 - 6x - 7}{x^2 - 6x + 5}
\]

Compute \(f(x)g(x)\) and simplify your answer.