7.3.3: Boundary Value Problems: Mixed Boundary Value Problem

The Mixed boundary value problem (third boundary value problem) is to find a solution \((u \in C^2(\Omega) \cap C^1(\partial \Omega))\) of
\[
\begin{align*}
\triangle u &= 0 \quad \text{in} \quad \Omega \\
\frac{\partial u}{\partial n} + hu &= \Phi \quad \text{on} \quad \partial \Omega,
\end{align*}
\]
where \(\Phi\) and \(h\) are given and continuous on \(\partial \Omega\).

Proposition 7.6. Assume \((\Omega)\) is bounded and sufficiently regular, then a solution to the mixed problem is uniquely determined in the class \((u \in C^2(\overline{\Omega}))\) provided \((h(x) \geq 0)\) on \((\partial \Omega)\) and \((h(x) > 0)\) for at least one point \((x \in \partial \Omega)\).

Proof. Exercise. Hint: Multiply the differential equation \((\triangle w = 0)\) by \((w)\) and integrate the result over \((\Omega)\).

Contributors

- Prof. Dr. Erich Miersemann (Universität Leipzig)