7.3.3: Boundary Value Problems: Mixed Boundary Value Problem

The Mixed boundary value problem (third boundary value problem) is to find a solution \( u \in C^2(\Omega) \cap C^1(\overline{\Omega}) \) of
\[
\begin{align*}
\triangle u &= 0 & \text{in} & & \Omega \label{M1} \tag{7.3.3.1} \\
\frac{\partial u}{\partial n} + hu &= \Phi & \text{on} & & \partial \Omega, \label{M2} \tag{7.3.3.2}
\end{align*}
\]
where \( \Phi \) and \( h \) are given and continuous on \( \partial \Omega \).

**Proposition 7.6.** Assume \( \Omega \) is bounded and sufficiently regular, then a solution to the mixed problem is uniquely determined in the class \( u \in C^2(\overline{\Omega}) \) provided \( h(x) \geq 0 \) on \( \partial \Omega \) and \( h(x) > 0 \) for at least one point \( x \in \partial \Omega \).

**Proof.** Exercise. Hint: Multiply the differential equation \( \triangle w = 0 \) by \( w \) and integrate the result over \( \Omega \).

**Contributors**

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