7.3.3: Boundary Value Problems: Mixed Boundary Value Problem

The *Mixed boundary value problem* (third boundary value problem) is to find a solution \(u \in C^2(\Omega) \cap C^1(\overline{\Omega})\) of

\[
\begin{align}
\triangle u &= 0 \quad \text{in} \quad \Omega \tag{7.3.3.1} \\
\frac{\partial u}{\partial n} + hu &= \Phi \quad \text{on} \quad \partial \Omega, \tag{7.3.3.2}
\end{align}
\]

where \(\Phi\) and \(h\) are given and continuous on \(\partial \Omega\).

**Proposition 7.6.** Assume \(\Omega\) is bounded and sufficiently regular, then a solution to the mixed problem is uniquely determined in the class \(u \in C^2(\overline{\Omega})\) provided \(h(x) \geq 0\) on \(\partial \Omega\) and \(h(x) > 0\) for at least one point \(x\in\partial\Omega\).

**Proof.** Exercise. Hint: Multiply the differential equation \(\triangle w = 0\) by \(w\) and integrate the result over \(\Omega\).

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