3.1: Mathematical Expressions

Recall the definition of a variable presented in Section 1.6.

Definition: Variable

A variable is a symbol (usually a letter) that stands for a value that may vary.

Let’s add the definition of a mathematical expression.

Definition: Mathematical Expression

When we combine numbers and variables in a valid way, using operations such as addition, subtraction, multiplication, division, exponentiation, and other operations and functions as yet unlearned, the resulting combination of mathematical symbols is called a mathematical expression.

Thus,

\[ 2a, x + 5, \text{ and } y^2, \]

being formed by a combination of numbers, variables, and mathematical operators, are valid mathematical expressions. A mathematical expression must be well-formed. For example,

\[ 2 + \div 5x \]

is not a valid expression because there is no term following the plus sign (it is not valid to write \(+\) with nothing between these operators). Similarly,
2 + 3(2)

is not well-formed because parentheses are not balanced.

Translating Words into Mathematical Expressions

In this section we turn our attention to translating word phrases into mathematical expressions. We begin with phrases that translate into sums. There is a wide variety of word phrases that translate into sums. Some common examples are given in Table \(\PageIndex{1a}\), though the list is far from complete. In like manner, a number of phrases that translate into differences are shown in Table \(\PageIndex{1b}\).

<table>
<thead>
<tr>
<th>Phrase</th>
<th>Translates to:</th>
<th>Phrase</th>
<th>Translates to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum of (x) and 12</td>
<td>(x + 12)</td>
<td>difference of (x) and 12</td>
<td>(x - 12)</td>
</tr>
<tr>
<td>4 greater than (b)</td>
<td>(b + 4)</td>
<td>4 less than (b)</td>
<td>(b - 4)</td>
</tr>
<tr>
<td>6 more than (y)</td>
<td>(y + 6)</td>
<td>7 subtracted from (y)</td>
<td>(y - 7)</td>
</tr>
<tr>
<td>44 plus (r)</td>
<td>(44 + r)</td>
<td>44 minus (r)</td>
<td>(44 - r)</td>
</tr>
<tr>
<td>3 larger than (z)</td>
<td>(z + 3)</td>
<td>3 smaller than (z)</td>
<td>(z - 3)</td>
</tr>
</tbody>
</table>

a) Phrases that are sums

b) Phrases that are differences

Let’s look at some examples, some of which translate into expressions involving sums, and some of which translate into expressions involving differences.

Example 1

Translate the following phrases into mathematical expressions:

a. “12 larger than \(x\),”

b. “11 less than \(y\),” and

c. “\(r\) decreased by 9.”

Solution

Here are the translations.

a. “12 larger than \(x\)” becomes \(x + 12\).

b. “11 less than \(y\)” becomes \(y - 11\).

c. “\(r\) decreased by 9” becomes \(r - 9\).
Exercise

Translate the following phrases into mathematical expressions:

a. “13 more than $x$” and
b. “12 fewer than $y$”.

Answer

(a) $x + 13$ and

(b) $y - 12$

Example 2

Let $W$ represent the width of the rectangle. The length of a rectangle is 4 feet longer than its width. Express the length of the rectangle in terms of its width $W$.

Solution

We know that the width of the rectangle is $W$. Because the length of the rectangle is 4 feet longer than the width, we must add 4 to the width to find the length.

$\begin{array}{c c c c c} \text{Length} & \text{is} & 4 & \text{more than} & \text{the width} \\ \text{Length} & = & 4 & + & W \end{array}$

Thus, the length of the rectangle, in terms of its width $W$, is $4 + W$.

Exercise

The width of a rectangle is 5 inches shorter than its length $L$. Express the width of the rectangle in terms of its length $L$.

Answer

$L - 5$

Example 3

A string measures 15 inches is cut into two pieces. Let $x$ represent the length of one of the resulting pieces. Express the length of the second piece in terms of the length $x$ of the first piece.

Solution

The string has original length 15 inches. It is cut into two pieces and the first piece has length $x$. To find the length of the second piece, we must subtract the length of the first piece from the total length.
Thus, the length of the second piece, in terms of the length $x$ of the first piece, Answer: $12 + x$ is $15 - x$.

Exercise

A string is cut into two pieces, the first of which measures 12 inches. Express the total length of the string as a function of $x$, where $x$ represents the length of the second piece of string.

**Answer**

$12 + x$

There is also a wide variety of phrases that translate into products. Some examples are shown in Table 3.2(a), though again the list is far from complete. In like manner, a number of phrases translate into quotients, as shown in Table 3.2(b).

Table \(\PageIndex{2}\): Translating words into symbols.

<table>
<thead>
<tr>
<th>Phrase</th>
<th>Translates to:</th>
<th>Phrase</th>
<th>Translates to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>product of $x$ and 12</td>
<td>$12x$</td>
<td>quotient of $x$ and 12</td>
<td>$x/12$</td>
</tr>
<tr>
<td>4 times $b$</td>
<td>$4b$</td>
<td>4 divided by $b$</td>
<td>$4/b$</td>
</tr>
<tr>
<td>twice $r$</td>
<td>$2r$</td>
<td>the ratio of 44 to $r$</td>
<td>$44/r$</td>
</tr>
</tbody>
</table>

a) Phrases that are products.  
b) Phrases that are differences.

Let’s look at some examples, some of which translate into expressions involving products, and some of which translate into expressions involving quotients.

Example 4

Translate the following phrases into mathematical expressions: (a) “11 times $x$,” (b) “quotient of $y$ and 4,” and (c) “twice $a$.”

**Solution**

Here are the translations. a) “11 times $x$” becomes $11x$. b) “quotient of $y$ and 4” becomes $y/4$, or equivalently, \(\frac{y}{4}\). c) “twice $a$” becomes $2a$.

Exercise

Translate into mathematical symbols: (a) “the product of 5 and $x$” and (b) “12 divided by $y$.”
\textbf{Example 5}

A plumber has a pipe of unknown length \( x \). He cuts it into 4 equal pieces. Find the length of each piece in terms of the unknown length \( x \).

\textbf{Solution}

The total length is unknown and equal to \( x \). The plumber divides it into 4 equal pieces. To find the length of each piece, we must divide the total length by 4.

\[
\begin{array}{c c c c c}
\text{Length of each piece} & \text{is} & \text{Total length} & \text{divided by} & 4 \\
\text{Length of each piece} & = & x & \div & 4
\end{array}
\]

Thus, the length of each piece, in terms of the unknown length \( x \), is \( x/4 \), or equivalently, \( \frac{x}{4} \).

\textbf{Exercise}

A carpenter cuts a board of unknown length \( L \) into three equal pieces. Express the length of each piece in terms of \( L \).

\textbf{Answer}

\( \frac{L}{3} \)

\textbf{Example 6}

Mary invests \( A \) dollars in a savings account paying 2\% interest per year. She invests five times this amount in a certificate of deposit paying 5\% per year. How much does she invest in the certificate of deposit, in terms of the amount \( A \) in the savings account?

\textbf{Solution}

The amount in the savings account is \( A \) dollars. She invests five times this amount in a certificate of deposit.

\[
\begin{array}{c c c c c}
\text{Amount in CD} & \text{is} & 5 & \text{times} & \text{Amount in savings} \\
\text{Amount in CD} & = & 5 & \cdot & A
\end{array}
\]

Thus, the amount invested in the certificate of deposit, in terms of the amount \( A \) in the savings account, is \( 5A \).

\textbf{Exercise}

David invests \( K \) dollars in a savings account paying 3\% per year. He invests half this amount in a mutual fund paying 4\% per
year. Express the amount invested in the mutual fund in terms of \( K \), the amount invested in the savings account.

**Answer**

\[
\frac{1}{2}K
\]

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**Combinations**

Some phrases require combinations of the mathematical operations employed in previous examples.

**Example 7**

Let the first number equal \( x \). The second number is 3 more than twice the first number. Express the second number in terms of the first number \( x \).

**Solution**

The first number is \( x \). The second number is 3 more than twice the first number.

\[
\begin{aligned}
\text{Second number} & \text{ is } 3 & \text{ more than } & \text{ twice the first number} \\
\text{Second number} & = & 3 & + & 2x
\end{aligned}
\]

Therefore, the second number, in terms of the first number \( x \), is \( 3 + 2x \).

**Exercise**

A second number is 4 less than 3 times a first number. Express the second number in terms of the first number \( y \).

**Answer**

\[3y - 4\]

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**Example 8**

The length of a rectangle is \( L \). The width is 15 feet less than 3 times the length. What is the width of the rectangle in terms of the length \( L \)?

**Solution**

The length of the rectangle is \( L \). The width is 15 feet less than 3 times the length.

\[
\begin{aligned}
\text{Width} & \text{ is } 3 & \text{ times the length} & \text{ less } & 15 \\
\text{Width} & = & 3L & - & 15
\end{aligned}
\]

Therefore, the width, in terms of the length \( L \), is \( 3L - 15 \).
Exercise

The width of a rectangle is $W$. The length is 7 inches longer than twice the width. Express the length of the rectangle in terms of its length $L$.

**Answer**

$$2W + 7$$

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**Exercises**

In Exercises 1-20, translate the phrase into a mathematical expression involving the given variable.

1. “8 times the width n”
2. “2 times the length z”
3. “6 times the sum of the number n and 3”
4. “10 times the sum of the number n and 8”
5. “the demand b quadrupled”
6. “the supply y quadrupled”
7. “the speed y decreased by 33”
8. “the speed u decreased by 30”
9. “10 times the width n”
10. “10 times the length z”
11. “9 times the sum of the number z and 2”
12. “14 times the sum of the number n and 10”
13. “the supply y doubled”
14. “the demand n quadrupled”
15. “13 more than 15 times the number p”
16. “14 less than 5 times the number y”
17. “4 less than 11 times the number x”
18. “13 less than 5 times the number p”
19. “the speed u decreased by 10”
20. “the speed w increased by 32”

21. Representing Numbers. Suppose n represents a whole number.
   i) What does n + 1 represent?
   ii) What does n + 2 represent?
   iii) What does n − 1 represent?

22. Suppose 2n represents an even whole number. How could we represent the next even number after 2n?
23. Suppose 2n + 1 represents an odd whole number. How could we represent the next odd number after 2n + 1?

24. There are b bags of mulch produced each month. How many bags of mulch are produced each year?

25. Steve sells twice as many products as Mike. Choose a variable and write an expression for each man’s sales.

26. Find a mathematical expression to represent the values.
   i) How many quarters are in d dollars?
   ii) How many minutes are in h hours?
   iii) How many hours are in d days?
   iv) How many days are in y years?
   v) How many months are in y years?
   vi) How many inches are in f feet?
   vii) How many feet are in y yards?

Answers

1. 8n
3. 6(n + 3)
5. $4b$

7. $y - 33$

9. $10n$

11. $9(z + 2)$

13. $2y$

15. $15p + 13$

17. $11x - 4$

19. $u - 10$

21.

  i) $n+1$ represents the next whole number after $n$.

  ii) $n+2$ represents the next whole number after $n + 1$, or, two whole numbers after $n$.

  iii) $n - 1$ represents the whole number before $n$.

23. $2n + 3$