2.6 Arguments and Rules of Inference

In this section we will look at how to test if an argument is valid. This is a test for the structure of the argument. A valid argument does not always mean you have a true conclusion; rather, the conclusion of a valid argument must be true if all the premises are true. We will also look at common valid arguments, known as Rules of Inference as well as common invalid arguments, known as Fallacies.

Arguments

Definition

An argument is a set of initial statements, called premises, followed by a conclusion.

Definition

An argument is valid if and only if in every case where all the premises are true, the conclusion is true. Otherwise, the argument is invalid.
Here is an example:

If I read my text, I will understand how to do my homework.

I understand how to do my homework.

Therefore, I read my text.

Our first premise: is If I read my text, then I understand how to do my homework.

Our second premise is: I understand how to do my homework.

Our conclusion is I read my text.

Let's use $t$ means I read my text and $u$ means I understand how to do my homework.

Symbolically, our argument is:

\[
(t \rightarrow u)
\]
\[
(u)
\]
\[
(\therefore t)
\]

**Testing the validity of an argument by truth table.**

We represent this argument by working out its premises and conclusion on a truth table:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$u$</th>
<th>$(t \rightarrow u)$</th>
<th>$(u)$</th>
<th>$(\therefore t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Notice we repeat the column for \(u\) and the column for \(t\) because one is a premise and one is a conclusion.

Since a valid argument must have a true conclusion in all cases where the premises are true, we need to examine the rows where all premises are true.

**Definition**

Given a truth table representing an argument, the rows where all the premises are true are called the **critical rows**.

We test an argument by considering all the critical rows. If the conclusion is true in all critical rows, then the argument is valid. This is another way of saying the conclusion of a valid argument must be true in every case where all the premises are true.

Look for rows where all premises are true.

<table>
<thead>
<tr>
<th>(\neg t)</th>
<th>(\neg u)</th>
<th>(\neg (t \rightarrow u))</th>
<th>(\neg u)</th>
<th>(\neg t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

We see that the 1st and 3rd rows are critical rows. In the 1st row, the conclusion is true. However, in the 3rd row, a critical row, the conclusion is false.

Thus this argument is ________________.

**Answer**

INVALID
Example

Consider this argument.

*If* Pat goes to the store, *Pat will buy* $1,000,000 *worth of food.*

*Pat goes to the store.*

*Therefore, Pat buys* $1,000,000 *worth of food.*

This is a valid argument (you can test it on a truth table).

However, even though Pat goes to the store, Pat does not buy $1,000,000 worth of food. The conclusion is false.

How can the conclusion of a valid argument be false?

**Solution**

The validity of an argument refers to its structure. Given a valid argument, the conclusion must be true if the premises are true. In this case the first premise is NOT true, and thus the conclusion does not need to be true.

The conclusion of a valid argument can be false if one or more of the premises is false.

**Rules of Inference**

A number of valid arguments are very common and are given names. Know these four:

**Modus Ponens**

\( \left( p \rightarrow q \right) \)

\( p \)

\( \therefore q \)
**Modus Tollens**

\[(p \rightarrow q)\]

\[\sim(q)\]

\[\therefore \sim(p)\]

**Elimination**

\[(p \lor q)\]

\[\sim(p)\]

\[\therefore q\]

**Transitivity**

\[(p \rightarrow q)\]

\[(q \rightarrow r)\]

\[\therefore p \rightarrow r\]

As you think about the rules of inference above, they should make sense to you. Furthermore, each one can be proved by a truth table.

If you see an argument in the form of a rule of inference, you know it's valid.

Example (PageIndex{2})

Explain why this argument is valid:

*If I go to the movies, I will not do my homework.*

*I do my homework.*

*Therefore, I did not go to the movies.*
Solution

This is valid by Modus Tollens.

Fallacies

Fallacies are invalid arguments. Know the names of these two common fallacies.

Converse Error

\[(p \rightarrow q)\]
\[(q)\]
\[\therefore p\]

Inverse Error

\[(p \rightarrow q)\]
\[\neg p\]
\[\therefore \neg q\]

If you think about the converse and inverse (and that they do not have the same meaning as the original implication) you can see why these fallacies have these names. You can use a truth table to show these fallacies are arguments that are\_______________.

Answer

INVALID

Example \(\PageIndex{3}\)
Explain why this argument is valid or invalid:

If I go to the movies, I will not do my homework.

I did not go to the movies.

Therefore, I did do my homework.

**Solution**

This is invalid; it is an inverse error.

---

**Exercises** \(\PageIndex{}\)

Exercise \(\PageIndex{1}\)

True or False?

(a) Given a valid argument with true premises, the conclusion must be true.

(b) Given a valid argument with false premises, the conclusion must be false.

(c) Given an invalid argument, the conclusion must be false.

**Answer**

(a) true  (b) false  (c) false

Exercise \(\PageIndex{3}\)

Decide if the following arguments are valid or invalid. State the Rule of Inference of fallacy used.

(a)

If it snows, then school is closed.

School is open.

Therefore it is not snowing.
(b)

My pet is a cat or my pet is a dog.
My pet is not a dog.
Therefore my pet is a cat.

(c)

If the movie is long, I will fall asleep.
I do fall asleep.
Therefore the movie was long.

**Answer**

(a) VALID, Modus Tollens
(b) VALID, Elimination
(c) INVALID, Converse Error

Exercise \(\PageIndex{5}\)

Use a truth table to determine if this argument is valid or invalid. Include a clear explanation.

\[
\overline{p} \vee (q \rightarrow r) \\
\overline{r} \\
\therefore \overline{p \wedge q}
\]

**Answer**

As seen below, there are three critical rows, namely the 4th, 6th and 8th rows. We can see that in every case where all the premises are true, the conclusion is also true. Thus, this is a valid argument.
<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(r)</th>
<th>(\overline{p})</th>
<th>(\neg (p \rightarrow r))</th>
<th>(\neg (q \rightarrow r))</th>
<th>(\overline{p} \lor (q \rightarrow r))</th>
<th>(\overline{r})</th>
<th>(p \land q)</th>
<th>(\overline{p} \land \overline{q})</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

UC Davis ChemWiki is licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 United States License.
Exercise \(\PageIndex{7}\)

Use a truth table and an explanation to prove Modus Ponens is a valid form of an argument.

**Answer**

As seen below, the only critical row is the first row. We can see that in the one case that all the premises are true, the conclusion is also true. Thus, Modus Ponens has the form of a valid argument.

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(p \rightarrow q)</th>
<th>(p)</th>
<th>(q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>