1.5: Union, Intersection, Difference

Just as numbers are combined with operations such as addition, subtraction and multiplication, there are various operations that can be applied to sets. The Cartesian product (defined in Section 1.2) is one such operation; given sets \(A\) and \(B\), we can combine them with \(\times\) to get a new set \(A \times B\). Here are three new operations called union, intersection and difference.

Definition 1.5: Union, Intersection, and Difference

Suppose \(A\) and \(B\) are sets.

- The **union** of \(A\) and \(B\) is the set \(A \cup B = \{x : \ x \in A\} \text{ or } \{x \in B\}\).
- The **intersection** of \(A\) and \(B\) is the set \(A \cap B = \{x : \ x \in A\} \text{ and } \{x \in B\}\).
- The **difference** of \(A\) and \(B\) is the set \(A - B = \{x : \ x \in A\} \text{ and } \\{x \notin B\}\).

In words, the union \(A \cup B\) is the set of all things that are in \(A\) or in \(B\) (or in both). The intersection \(A \cap B\) is the set of all things in both \(A\) and \(B\). The difference \(A - B\) is the set of all things that are in \(A\) but not in \(B\).

Example 1.8

Suppose \(A = \{a,b,c,d,e\}\), \(B = \{d,e,f\}\) and \(C = \{1,2,3\}\).

1. \(A \cup B = \{a,b,c,d,e,f\}\)
2. \(A \cap B = \{d,e\}\)
3. \(A - B = \{a,b,c\}\)
4. \(B - A = \{f\}\)
5. \((A - B) \cup (B - A) = \{a,b,c,f\}\)
6. \(A \cup C = \{a,b,c,d,e,1,2,3\}\)
7. \(A \cap C = \{\emptyset\}\)
8. \((A - C) = \{a,b,c,d,e\}\)
9. \(((A \cap C) \cup (A-C)) = \{a,b,c,d,e\}\)
10. \(((A \cap B) \times B = \{(d,d),(d,e),(d,f),(e,d),(e,e),(e,f)\}\)
11. \(((A \times C) \cup (B \times C) = \{(d,1),(d,2),(d,3),(e,1),(e,2),(e,3)\}\)

Parts 12–15 use interval notation (Section 1.1), so \([2, 5] = \{x \in \mathbb{R} : 2 \le x \le 5\}\), etc. Sketching these on the number line may aid your understanding.

12. \([2, 5] \cup [3, 6] = [2, 6]\)
13. \([2, 5] \cap [3, 6] = [3, 5]\)
14. \([2, 5] - [3, 6] = [2, 3]\)
15. \([0, 3] - [1, 2] = [0, 1] \cup (2, 3]\)

Observe that for any sets X and Y it is always true that \(X \cup Y = Y \cup X\) and \(X \cap Y = Y \cap X\), but in general \(X - Y \ne Y - X\).

Example 1.9

Let \(A = \{(x, x^2) : x \in \mathbb{R}\}\) be the graph of the equation \(y = x^2\) and let \(B = \{(x, x+2) : x \in \mathbb{R}\}\) be the graph of the equation \(y = x+2\). These sets are subsets of \(\mathbb{R}^2\). They are sketched together in Figure 1.5(a). Figure 1.5(b) shows \(A \cup B\), the set of all points \((x, y)\) that are on one (or both) of the two graphs. Observe that \(A \cap B = \{(−1, 1), (2, 4)\}\) consists of just two elements, the two points where the graphs intersect, as illustrated in Figure 1.5(c). Figure 1.5(d) shows \(A - B\), which is the set \(A\) with "holes" where \(B\) crossed it. In set builder notation, we could write \(A \cup B = \{(x, y) : x \in \mathbb{R}, y = x^2\text{ or } y = x+2\}\) or \(\{(y = x^2) : x \in \mathbb{R}\}\) and \(A - B = \{(x, x^2) : x \in \mathbb{R} - \{-1, 2\}\}\).

![Figure 1.5. The union, intersection and difference of sets A and B](image)

Exercise

Exercise \(\PageIndex{1}\)

Suppose \(A = \{4,3,6,7,1,9\}\), \(B = \{5,6,8,4\}\) and \(C = \{5,8,4\}\). Find:

a. \(A \cup B\)

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Exercise \(\PageIndex{2}\)

Suppose \(A = \{0,2,4,6,8\}\), \(B = \{1,3,5,7\}\) and \(C = \{2,8,4\}\). Find:

a. \(A \cup B\)

b. \(A \cap B\)

c. \(A-B\)

d. \(A-C\)

e. \(B-A\)

f. \(A \cap C\)

g. \(B \cap C\)

h. \(B \cup C\)

i. \(C-B\)

Exercise \(\PageIndex{3}\)

Suppose \(A = \{0,1\}\) and \(B = \{1,2\}\). Find:

a. \((A \times B) \cap (B \times B)\)

b. \((A \times B) \cup (B \times B)\)

c. \((A \times B) - (B \times B)\)

d. \((A \cap B) \times A\)

e. \((A \times B) \cap B\)

f. \(\mathscr{P}(A) \cap \mathscr{P}(B)\)

g. \(\mathscr{P}(A) - \mathscr{P}(B)\)

h. \(\mathscr{P}(A \cap B)\)

i. \(\mathscr{P}(A) \times \mathscr{P}(B)\)

Exercise \(\PageIndex{4}\)

Suppose \(A = \{b,c,d\}\) and \(B = \{a,b\}\). Find:

a. \((A \times B) \cap (B \times B)\)
b. \((A \times B) \cup (B \times B)\)

c. \((A \times B) \setminus (B \times B)\)

d. \((A \setminus B) \times A\)

e. \((A \times B) \cap B\)

f. \(\mathscr{P}(A) \cap \mathscr{P}(B)\)

g. \(\mathscr{P}(A) \cup \mathscr{P}(B)\)

h. \(\mathscr{P}(A) \setminus \mathscr{P}(B)\)

i. \(\mathscr{P}(A) \times \mathscr{P}(B)\)

Exercise \(\PageIndex{5}\)

Sketch the sets \(X = [1, 3] \times [1, 3]\) and \(Y = [2,4] \times [2,4]\) on the plane \(\mathbb{R}^2\). On separate drawings, shade in the sets \(X \cup Y\), \(X \cap Y\), \((X - Y)\) and \((Y - X)\). (Hint: \(X\) and \(Y\) are Cartesian products of intervals. You may wish to review how you drew sets like \([1,3] \times [1,3]\) in the exercises for Section 1.2.)

Exercise \(\PageIndex{6}\)

Sketch the sets \(X = [-1, 3] \times [0, 2]\) and \(Y = [0, 3] \times [1,4]\) on the plane \(\mathbb{R}^2\). On separate drawings, shade in the sets \(X \cup Y\), \(X \cap Y\), \((X - Y)\) and \((Y - X)\).

Exercise \(\PageIndex{7}\)

Sketch the sets \(X = \{(x, y) \in \mathbb{R}^2 : x^2+y^2 \le 1\}\) and \(Y = \{(x, y) \in \mathbb{R}^2 : x \ge 0\}\) on the plane \(\mathbb{R}^2\). On separate drawings, shade in the sets \(X \cup Y\), \(X \cap Y\), \((X - Y)\) and \((Y - X)\).

Exercise \(\PageIndex{8}\)

Sketch the sets \(X = \{(x, y) \in \mathbb{R}^2 : x^2+y^2 \le 1\}\) and \(Y = \{(x, y) \in \mathbb{R}^2 : x \ge 0\}\) on the plane \(\mathbb{R}^2\). On separate drawings, shade in the sets \(X \cup Y\), \(X \cap Y\), \((X - Y)\) and \((Y - X)\).

Exercise \(\PageIndex{9}\)

Is the statement \((\mathbb{R} \times \mathbb{Z}) \cap (\mathbb{Z} \times \mathbb{R}) = \mathbb{Z} \times \mathbb{Z}\) true or false? What about the statement \((\mathbb{R} \times \mathbb{Z}) \cup (\mathbb{Z} \times \mathbb{R}) = \mathbb{R} \times \mathbb{R}\)?

Exercise \(\PageIndex{10}\)

Do you think the statement \((\mathbb{R} \setminus \mathbb{Z}) \times \mathbb{N} = (\mathbb{R} \times \mathbb{N}) \setminus (\mathbb{Z} \times \mathbb{N})\) is true, or false? Justify.