2.5: Solve Mixture and Uniform Motion Applications

Learning Objectives

By the end of this section, you will be able to:

- Solve coin word problems
- Solve ticket and stamp word problems
- Solve mixture word problems
- Solve uniform motion applications

Before you get started, take this readiness quiz.

1. Simplify: \(0.25x+0.10(x+4)\).
   If you missed this problem, review [link].

2. The number of adult tickets is three more than twice the number of children tickets. Let \(c\) represent the number of children tickets. Write an expression for the number of adult tickets.
   If you missed this problem, review [link].

3. Convert 4.2% to a decimal.
   If you missed this problem, review [link].

Solve Coin Word Problems

Using algebra to find the number of nickels and pennies in a piggy bank may seem silly. You may wonder why we just don’t open the bank and count them. But this type of problem introduces us to some techniques that will be useful as we move forward in our study of mathematics.
If we have a pile of dimes, how would we determine its value? If we count the number of dimes, we’ll know how many we have—the number of dimes. But this does not tell us the value of all the dimes. Say we counted 23 dimes, how much are they worth? Each dime is worth $0.10—that is the value of one dime. To find the total value of the pile of 23 dimes, multiply 23 by $0.10 to get $2.30.

The number of dimes times the value of each dime equals the total value of the dimes.

\[
\begin{align}
\text{number} \cdot \text{value} &= \text{total value} \\
23 \cdot $0.10 &= $2.30
\end{align}
\]

This method leads to the following model.

TOTAL VALUE OF COINS

For the same type of coin, the total value of a number of coins is found by using the model

\[
\text{number} \cdot \text{value} = \text{total value}
\]

- number is the number of coins
- value is the value of each coin
- total value is the total value of all the coins

If we had several types of coins, we could continue this process for each type of coin, and then we would know the total value of each type of coin. To get the total value of all the coins, add the total value of each type of coin.

Example

Jesse has $3.02 worth of pennies and nickels in his piggy bank. The number of nickels is three more than eight times the number of pennies. How many nickels and how many pennies does Jesse have?

Answer

Step 1. Read the problem.
Determine the types of coins involved. pennies and nickels
Create a table. Write in the value of each type of coin.

Pennies are worth $0.10. Nickels are worth $0.05.

**Step 2. Identify** what we are looking for.

the number of pennies and nickels

**Step 3. Name.** Represent the number of each type of coin using variables.

The number of nickels is defined in terms of the number of pennies, so start with pennies.

The number of nickels is three more than eight times the number of pennies.

Let \(p\) number of pennies. \(8p+3\) number of nickels

In the chart, multiply the number and the value to get the total value of each type of coin.

<table>
<thead>
<tr>
<th>Type</th>
<th>Number • Value ($) = Total Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pennies</td>
<td>(p) • 0.01 = 0.01(p)</td>
</tr>
<tr>
<td>nickels</td>
<td>(8p + 3) • 0.05 = 0.05(8p + 3)</td>
</tr>
</tbody>
</table>

$3.02$

**Step 4. Translate.** Write the equation by adding the total value of all the types of coins.

\[
0.01p + 0.05(8p + 3) = 3.02
\]

**Step 5. Solve** the equation.

\[
0.01p + 0.40p + 0.15 = 3.02
\]

\[
0.41p + 0.15 = 3.02
\]

\[
0.41p = 2.87
\]

\[
p = 7 \text{ pennies}
\]

How many nickels?

\[
8p + 3
\]

\[
8(7) + 3
\]

59 nickels

**Step 6. Check** the answer in the problem and make sure it makes sense.

Jesse has 7 pennies and 59 nickels.

Is the total value $3.02$?

\[
\begin{align*}
\text{begin{align*} 7(0.01) + 59(0.05) &\overset{?}{=} 3.02 \\
\text{nonumber} \ &\text{\hfill} 3.02 \text{ \checkmark \ nonumber} \\
\text{end{align*}}
\end{align*}
\]
Example \(\PageIndex{2}\)

Jesse has $6.55 worth of quarters and nickels in his pocket. The number of nickels is five more than two times the number of quarters. How many nickels and how many quarters does Jesse have?

**Answer**

Jess has 41 nickels and 18 quarters.

Example \(\PageIndex{3}\)

Elane has $7.00 total in dimes and nickels in her coin jar. The number of dimes that Elane has is seven less than three times the number of nickels. How many of each coin does Elane have?

**Answer**

Elane has 22 nickels and 59 dimes.

The steps for solving a coin word problem are summarized below.

**SOLVE COIN WORD PROBLEMS.**

1. **Read** the problem. Make sure all the words and ideas are understood.
   - Determine the types of coins involved.
   - Create a table to organize the information.
     - Label the columns “type,” “number,” “value,” and “total value.”
     - List the types of coins.
     - Write in the value of each type of coin.
     - Write in the total value of all the coins.

<table>
<thead>
<tr>
<th>Type</th>
<th>Number</th>
<th>Value ($)</th>
<th>Total Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. **Identify** what you are looking for.

3. **Name** what you are looking for. Choose a variable to represent that quantity.
   - Use variable expressions to represent the number of each type of coin and write them in the table.
   - Multiply the number times the value to get the total value of each type of coin.

4. **Translate** into an equation.
   - It may be helpful to restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.
   - Write the equation by adding the total values of all the types of coins.

5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

---

**Solve Ticket and Stamp Word Problems**

Problems involving tickets or stamps are very much like coin problems. Each type of ticket and stamp has a value, just like each type of coin does. So to solve these problems, we will follow the same steps we used to solve coin problems.

Example \( \PageIndex{4} \)

Danny paid $15.75 for stamps. The number of 49-cent stamps was five less than three times the number of 35-cent stamps. How many 49-cent stamps and how many 35-cent stamps did Danny buy?

**Answer**

**Step 1.** Determine the types of stamps involved.

**Step 2. Identify** we are looking for.

**Step 3.** Write variable expressions to represent the number of each type of stamp.

“The number of 49-cent stamps was five less than three times the number of 35-cent stamps.”

<table>
<thead>
<tr>
<th>Type</th>
<th>Number</th>
<th>Value($)</th>
<th>Total Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>49 cent stamps</td>
<td>(3x-5)</td>
<td>0.49</td>
<td>(0.49(3x-5))</td>
</tr>
<tr>
<td>35 cent stamps</td>
<td>(x)</td>
<td>0.35</td>
<td>(0.35x)</td>
</tr>
</tbody>
</table>

**Step 4.** Write the equation from the total values.

**Step 5. Solve** the equation.

How many 49-cent stamps?
Step 6. Check.
\[
10(0.35)+25(0.49)\quad 3.50+12.25\quad 15.75 = 15.75 \checkmark
\]

Step 7. Answer the question with a complete sentence.

Example \(\PageIndex{5}\)

Eric paid $19.88 for stamps. The number of 49-cent stamps was eight more than twice the number of 35-cent stamps. How many 49-cent stamps and how many 35-cent stamps did Eric buy?

Answer

Eric bought thirty-two 49-cent stamps and twelve 35-cent stamps.

Example \(\PageIndex{6}\)

Kailee paid $14.74 for stamps. The number of 49-cent stamps was four less than three times the number of 20-cent stamps. How many 49-cent stamps and how many 20-cent stamps did Kailee buy?

Answer

Kailee bought twenty-six 49-cent stamps and ten 20-cent stamps.

In most of our examples so far, we have been told that one quantity is four more than twice the other, or something similar. In our next example, we have to relate the quantities in a different way.

Suppose Aniket sold a total of 100 tickets. Each ticket was either an adult ticket or a child ticket. If he sold 20 child tickets, how many adult tickets did he sell?

Did you say “80”? How did you figure that out? Did you subtract 20 from 100?

If he sold 45 child tickets, how many adult tickets did he sell?

Did you say “55”? How did you find it? By subtracting 45 from 100?

Now, suppose Aniket sold \(x\) child tickets. Then how many adult tickets did he sell? To find out, we would follow the same logic we used above. In each case, we subtracted the number of child tickets from 100 to get the number of adult tickets. We now do the same with \(x\).

We have summarized this in the table.
We will apply this technique in the next example.

Example \(\PageIndex{7}\)

A whale-watching ship had 40 paying passengers on board. The total revenue collected from tickets was $1,196. Full-fare passengers paid $32 each and reduced-fare passengers paid $26 each. How many full-fare passengers and how many reduced-fare passengers were on the ship?

**Answer**

**Step 1.** Determine the types of tickets involved.

**Step 2. Identify** what we are looking for.

**Step 3. Name.** Represent the number of each type of ticket using variables.

We know the total number of tickets sold was 40. This means the number of reduced-fare tickets is 40 less the number of full-fare tickets. Multiply the number times the value to get the total value of each type of ticket.

**Step 4. Translate.** Write the equation by adding the total values of each type of ticket.

\[
32f + 26(40 - f) = 1,196
\]

**Step 5. Solve** the equation.

\[
32f + 1,040 - 26f = 1,196
\]

\[
f = 26 \quad \text{full-fare tickets}
\]
How many reduced-fare?

**Step 6. Check** the answer.
There were 26 full-fare tickets at $32 each and 14 reduced-fare tickets at $26 each. Is the total value $116?

\[26 \cdot 32 + 14 \cdot 26 = 832 + 364 = 1196\]

\[26 \cdot 32 = 832\]
\[14 \cdot 26 = 364\]

\[= 1,196\]

\[\checkmark\]

**Step 7. Answer** the question.
They sold 26 full-fare and 14 reduced-fare tickets.

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Example \(\PageIndex{8}\)

During her shift at the museum ticket booth, Leah sold 115 tickets for a total of $1,163. Adult tickets cost $12 and student tickets cost $5. How many adult tickets and how many student tickets did Leah sell?

**Answer**

84 adult tickets, 31 student tickets

Example \(\PageIndex{9}\)

Galen sold 810 tickets for his church’s carnival for a total revenue of $2,820. Children’s tickets cost $3 each and adult tickets cost $5 each. How many children’s tickets and how many adult tickets did he sell?

**Answer**

615 children’s tickets and 195 adult tickets

---

**Solve Mixture Word Problems**

Now we’ll solve some more general applications of the mixture model. In mixture problems, we are often mixing two quantities, such as raisins and nuts, to create a mixture, such as trail mix. In our tables we will have a row for each item to be mixed as well as one for the final mixture.

Example \(\PageIndex{10}\)

Henning is mixing raisins and nuts to make 25 pounds of trail mix. Raisins cost $4.50 a pound and nuts cost $8 a pound. If Henning wants his cost for the trail mix to be $6.60 a pound, how many pounds of raisins and how many pounds of nuts should he use?
Answer

Step 1. Determine what is being mixed.

Step 2. Identify what we are looking for.

Step 3. Represent the number of each type of ticket using variables.

As before, we fill in a chart to organize our information.
We enter the price per pound for each item.
We multiply the number times the value to get the total value.

<table>
<thead>
<tr>
<th>Type</th>
<th>Number of pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raisins</td>
<td>x</td>
</tr>
<tr>
<td>Nuts</td>
<td>25 - x</td>
</tr>
<tr>
<td>Trail mix</td>
<td>25</td>
</tr>
</tbody>
</table>

Notice that the last column in the table gives the information for the total amount of the mixture.

Step 4. Translate into an equation.

Step 5. Solve the equation.

Find the number of pounds of nuts.

Step 6. Check.

Step 7. Answer the question.

Example

Orlando is mixing nuts and cereal squares to make a party mix. Nuts sell for $7 a pound and cereal squares sell for $4 a pound. Orlando wants to make 30 pounds of party mix at a cost of $6.50 a pound, how many pounds of nuts and how many pounds of cereal squares should he use?

Answer

Orlando mixed five pounds of cereal squares and 25 pounds of nuts.
Example \(\PageIndex{12}\)

Becca wants to mix fruit juice and soda to make a punch. She can buy fruit juice for $3 a gallon and soda for $4 a gallon. If she wants to make 28 gallons of punch at a cost of $3.25 a gallon, how many gallons of fruit juice and how many gallons of soda should she buy?

**Answer**

Becca mixed 21 gallons of fruit punch and seven gallons of soda.

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### Solve Uniform Motion Applications

When you are driving down the interstate using your cruise control, the speed of your car stays the same—it is uniform. We call a problem in which the speed of an object is constant a **uniform motion** application. We will use the distance, rate, and time formula, \((D=rt)\), to compare two scenarios, such as two vehicles traveling at different rates or in opposite directions.

Our problem solving strategies will still apply here, but we will add to the first step. The first step will include drawing a diagram that shows what is happening in the example. Drawing the diagram helps us understand what is happening so that we will write an appropriate equation. Then we will make a table to organize the information, like we did for the coin, ticket, and stamp applications.

The steps are listed here for easy reference:

**SOLVE A UNIFORM MOTION APPLICATION.**

1. **Read** the problem. Make sure all the words and ideas are understood.
   - Draw a diagram to illustrate what is happening.
   - Create a table to organize the information.
     - Label the columns rate, time, distance.
     - List the two scenarios.
     - Write in the information you know.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. **Identify** what you are looking for.

3. **Name** what you are looking for. Choose a variable to represent that quantity.
   - Complete the chart.
   - Use variable expressions to represent that quantity in each row.
   - Multiply the rate times the time to get the distance.
4. **Translate** into an equation.
   ◦ Restate the problem in one sentence with all the important information.
   ◦ Then, translate the sentence into an equation.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Example \(\PageIndex{13}\)

Wayne and Dennis like to ride the bike path from Riverside Park to the beach. Dennis’s speed is seven miles per hour faster than Wayne’s speed, so it takes Wayne two hours to ride to the beach while it takes Dennis 1.5 hours for the ride. Find the speed of both bikers.

**Answer**

**Step 1. Read** the problem. Make sure all the words and ideas are understood.

Draw a diagram to illustrate what is happening. Shown below is a sketch of what is happening in the example.

![Diagram of the bike path from Riverside Park to the beach showing Dennis and Wayne's speeds and times](image)

Create a table to organize the information.

- Label the columns “Rate,” “Time,” and “Distance.”
- List the two scenarios.
- Write in the information you know.

<table>
<thead>
<tr>
<th></th>
<th>Rate (mph)</th>
<th>Time (hrs)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dennis</td>
<td></td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Wayne</td>
<td></td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

**Step 2. Identify** what you are looking for.

You are asked to find the speed of both bikers.

Notice that the distance formula uses the word “rate,” but it is more common to use “speed” when we talk about vehicles in everyday English.
Step 3. Name what we are looking for. Choose a variable to represent that quantity.

Complete the chart. Use variable expressions to represent that quantity in each row.
We are looking for the speed of the bikers. Let’s let $r$ represent Wayne’s speed. Since Dennis’ speed is 7 mph faster, we represent that as $(r+7)$.

\[
\begin{align*}
r + 7 &= \text{Dennis’ speed} \\
r &= \text{Wayne’s speed}
\end{align*}
\]

Fill in the speeds into the chart.

<table>
<thead>
<tr>
<th></th>
<th>Rate (mph)</th>
<th>Time (hrs)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dennis</td>
<td>$r + 7$</td>
<td>1.5</td>
<td>$1.5(r + 7)$</td>
</tr>
<tr>
<td>Wayne</td>
<td>$r$</td>
<td>2</td>
<td>$2r$</td>
</tr>
</tbody>
</table>

Multiply the rate times the time to get the distance.

Step 4. Translate into an equation.

Restate the problem in one sentence with all the important information.

Then, translate the sentence into an equation.

The equation to model this situation will come from the relation between the distances. Look at the diagram we drew above. How is the distance travelled by Dennis related to the distance travelled by Wayne?

Since both bikers leave from Riverside and travel to the beach, they travel the same distance. So we write:

\[
\text{distance traveled by Dennis} = \text{distance traveled by Wayne}
\]

Step 5. Solve the equation using algebra techniques.

\[
\begin{align*}
1.5(r + 7) &= 2r \\
1.5r + 10.5 &= 2r
\end{align*}
\]

Now solve this equation.

\[
\begin{align*}
10.5 &= 0.5r \\
21 &= r
\end{align*}
\]

So Wayne’s speed is 21 mph.
Find Dennis’ speed.

\[
\begin{align*}
21 + 7 &= 28 \\
\text{Dennis’ speed} &= 28 \text{ mph.}
\end{align*}
\]

**Step 6. Check** the answer in the problem and make sure it makes sense.

\[
\begin{align*}
28\text{ mph}(1.5\text{ hours}) &= 42\text{ miles} \checkmark \\
21\text{ mph}(2\text{ hours}) &= 42\text{ miles} \checkmark
\end{align*}
\]

**Step 7. Answer** the question with a complete sentence.

Wayne rode at 21 mph and Dennis rode at 28 mph.

Restate the problem in one sentence with all the important information.

Then, translate the sentence into an equation.

The equation to model this situation will come from the relation between the distances. Look at the diagram we drew above. How is the distance travelled by Dennis related to the distance travelled by Wayne?

Since both bikers leave from Riverside and travel to the beach, they travel the same distance. So we write:

\[
\text{distance traveled by Dennis} = \text{distance traveled by Wayne}
\]

\[
1.5(r + 7) = 2r
\]

Example \((\PageIndex{14})\)

An express train and a local train leave Pittsburgh to travel to Washington, D.C. The express train can make the trip in four hours and the local train takes five hours for the trip. The speed of the express train is 12 miles per hour faster than the speed of the local train. Find the speed of both trains.

**Answer**

The speed of the local train is 48 mph and the speed of the express train is 60 mph.

Example \((\PageIndex{15})\)

Jeromy can drive from his house in Cleveland to his college in Chicago in 4.5 hours. It takes his mother six hours to make the same drive. Jeromy drives 20 miles per hour faster than his mother. Find Jeromy’s speed and his mother’s speed.
In Example, we had two bikers traveling the same distance. In the next example, two people drive toward each other until they meet.

Example \((\PageIndex{16})\)

Carina is driving from her home in Anaheim to Berkeley on the same day her brother is driving from Berkeley to Anaheim, so they decide to meet for lunch along the way in Buttonwillow. The distance from Anaheim to Berkeley is 395 miles. It takes Carina three hours to get to Buttonwillow, while her brother drives four hours to get there. Carina’s average speed is 15 miles per hour faster than her brother’s average speed. Find Carina’s and her brother’s average speeds.

**Answer**

**Step 1. Read** the problem. Make sure all the words and ideas are understood.

Draw a diagram to illustrate what it happening. Below shows a sketch of what is happening in the example.

![Diagram showing Carina and her brother driving towards each other and meeting for lunch in Buttonwillow.](image)

Create a table to organize the information.

- Label the columns rate, time, distance.
- List the two scenarios.
- Write in the information you know.

<table>
<thead>
<tr>
<th></th>
<th>Rate (mph)</th>
<th>Time (hrs)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carina</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Brother</td>
<td></td>
<td>4</td>
<td>395</td>
</tr>
</tbody>
</table>

**Step 2. Identify** what we are looking for.

We are asked to find the average speeds of Carina and her brother.

**Step 3. Name** what we are looking for. Choose a variable to represent that quantity.

Complete the chart. Use variable expressions to represent that quantity in each row.
We are looking for their average speeds. Let’s let \( r \) represent the average speed of Carina. Since the brother’s speed is 15 mph faster, we represent that as \( r + 15 \).

Fill in the speeds into the chart. Multiply the rate times the time to get the distance.

<table>
<thead>
<tr>
<th></th>
<th>Rate (mph)</th>
<th>Time (hrs)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carina</td>
<td>( r + 15 )</td>
<td>3</td>
<td>( 3(r + 15) )</td>
</tr>
<tr>
<td>Brother</td>
<td>( r )</td>
<td>4</td>
<td>( 4r )</td>
</tr>
</tbody>
</table>

**Step 4. Translate** into an equation.

Restate the problem in one sentence with all the important information. Then, translate the sentence into an equation. Again, we need to identify a relationship between the distances in order to write an equation. Look at the diagram we created above and notice the relationship between the distance Carina traveled and the distance her brother traveled. The distance Carina traveled plus the distance her brother travel must add up to 410 miles. So we write:

\[
\text{distance traveled by Carina} + \text{distance traveled by her brother} = 395
\]

Translate to an equation:

\[
3(r + 15) + 4r = 395
\]

**Step 5. Solve** the equation using algebra techniques.

\[
3(r + 15) + 4r = 395
\]

\[
3r + 45 + 4r = 395
\]

\[
7r + 45 = 395
\]

Now solve this equation.

\[
7r = 350
\]

\[
r = 50
\]

So Carina’s brother's speed was 50 mph.

Carina's speed is \( r + 15 \).

\[
r + 15
\]

\[
50 + 15
\]

Her brother’s speed was 65 mph.

**Step 6. Check** the answer in the problem and make sure it makes sense.

\[
\begin{array}{llll}
\text{Carina drove} & 65 \text{ mph}(3 \text{ hours}) &=& \underline{195 \text{ miles}} \\
\text{Her brother drove} & 50 \text{ mph}(4 \text{ hours}) &=& \underline{200 \text{ miles}} \\
\end{array}
\]
Step 7. Answer the question with a complete sentence.

Carina drove 65 mph and her brother 50 mph.

Example \(\PageIndex{17}\)

Christopher and his parents live 115 miles apart. They met at a restaurant between their homes to celebrate his mother’s birthday. Christopher drove one and a half hours while his parents drove one hour to get to the restaurant. Christopher’s average speed was ten miles per hour faster than his parents’ average speed. What were the average speeds of Christopher and of his parents as they drove to the restaurant?

Answer

Christopher’s speed was 50 mph and his parents’ speed was 40 mph.

Example \(\PageIndex{18}\)

Ashley goes to college in Minneapolis, 234 miles from her home in Sioux Falls. She wants her parents to bring her more winter clothes, so they decide to meet at a restaurant on the road between Minneapolis and Sioux Falls. Ashley and her parents both drove two hours to the restaurant. Ashley’s average speed was seven miles per hour faster than her parents’ average speed. Find Ashley’s and her parents’ average speed.

Answer

Ashley’s parents drove 55 mph and Ashley drove 62 mph.

As you read the next example, think about the relationship of the distances traveled. Which of the previous two examples is more similar to this situation?

Example \(\PageIndex{19}\)

Two truck drivers leave a rest area on the interstate at the same time. One truck travels east and the other one travels west. The truck traveling west travels at 70 mph and the truck traveling east has an average speed of 60 mph. How long will they travel before they are 325 miles apart?

Answer

Step 1. Read the problem. Make all the words and ideas are understood.

Draw a diagram to illustrate what it happening.
Create a table to organize the information.

- Label the columns rate, time, distance.
- List the two scenarios.
- Write in the information you know.

<table>
<thead>
<tr>
<th></th>
<th>Rate (mph)</th>
<th>Time (hrs)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>West</td>
<td>70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>East</td>
<td>60</td>
<td></td>
<td>325</td>
</tr>
</tbody>
</table>

**Step 2. Identify** what we are looking for.

We are asked to find the amount of time the trucks will travel until they are 325 miles apart.

**Step 3. Name** what we are looking for. Choose a variable to represent that quantity.

Complete the chart. Use variable expressions to represent that quantity in each row.

We are looking for the time travelled. Both trucks will travel the same amount of time.

Let’s call the time \( t \). Since their speeds are different, they will travel different distances.

Multiply the rate times the time to get the distance.

<table>
<thead>
<tr>
<th></th>
<th>Rate (mph)</th>
<th>Time (hrs)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>West</td>
<td>70</td>
<td>( t )</td>
<td>70( t )</td>
</tr>
<tr>
<td>East</td>
<td>60</td>
<td>( t )</td>
<td>60( t )</td>
</tr>
</tbody>
</table>

**Step 4. Translate** into an equation.

Restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.

We need to find a relation between the distances in order to write an equation. Looking at the diagram, what is the relationship between the distances each of the trucks will travel?
The distance travelled by the truck going west plus the distance travelled by the truck going east must add up to 325 miles. So we write:

\[
\text{distance traveled by westbound truck} + \text{distance traveled by eastbound truck} = 325
\]

**Step 5. Solve** the equation using algebra techniques.

\[
\begin{align}
70t + 60t &= 325 \\
130t &= 325 \\
t &= 2.5
\end{align}
\]

So it will take the trucks \(2.5\) hours to be 325 miles apart.

**Step 6. Check** the answer in the problem and make sure it makes sense.

\[
\begin{array}{ll}
\text{Truck going West} & 70 \text{ mph}(2.5 \text{ hours}) = 175 \text{ miles} \\
\text{Truck going East} & 60 \text{ mph}(2.5 \text{ hours}) = 150 \text{ miles} \\
& 325 \text{ miles} \checkmark
\end{array}
\]

**Step 7. Answer** the question with a complete sentence.

It will take the trucks 2.5 hours to be 325 miles apart.

---

Example \(\PageIndex{20}\)

Pierre and Monique leave their home in Portland at the same time. Pierre drives north on the turnpike at a speed of 75 miles per hour while Monique drives south at a speed of 68 miles per hour. How long will it take them to be 429 miles apart?

**Answer**

Pierre and Monique will be 429 miles apart in 3 hours.

Example \(\PageIndex{21}\)

Thanh and Nhat leave their office in Sacramento at the same time. Thanh drives north on I-5 at a speed of 72 miles per hour. Nhat drives south on I-5 at a speed of 76 miles per hour. How long will it take them to be 330 miles apart?

**Answer**

Thanh and Nhat will be 330 miles apart in 2.2 hours.

It is important to make sure that the units match when we use the distance rate and time formula. For instance, if the rate is in miles per hour, then the time must be in hours.

Example \(\PageIndex{22}\)
When Naoko walks to school, it takes her 30 minutes. If she rides her bike, it takes her 15 minutes. Her speed is three miles per hour faster when she rides her bike than when she walks. What is her speed walking and her speed riding her bike?

**Answer**

First, we draw a diagram that represents the situation to help us see what is happening.

We are asked to find her speed walking and riding her bike. Let’s call her walking speed \( r \). Since her biking speed is three miles per hour faster, we will call that speed \( (r+3) \). We write the speeds in the chart.

The speed is in miles per hour, so we need to express the times in hours, too, in order for the units to be the same. Remember, 1 hour is 60 minutes. So:

\[
\begin{array}{l}
\text{30 minutes is } \frac{30}{60} \text{ or } \frac{1}{2} \text{ hour} \\
\text{15 minutes is } \frac{15}{60} \text{ or } \frac{1}{4} \text{ hour}
\end{array}
\]

We write the times in the chart.

Next, we multiply rate times time to fill in the distance column.

<table>
<thead>
<tr>
<th></th>
<th>Rate (mph)</th>
<th>Time (hrs)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walk</td>
<td>( r )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2}r )</td>
</tr>
<tr>
<td>Bike</td>
<td>( r+3 )</td>
<td>( \frac{1}{4} )</td>
<td>( \frac{1}{4}(r+3) )</td>
</tr>
</tbody>
</table>

The equation will come from the fact that the distance from Naoko’s home to her school is the same whether she is walking or riding her bike.

So we say:
\[
\begin{align*}
\text{distance walked} &= \text{distance covered by bike} \\
\frac{1}{2}r &= \frac{1}{4}(r + 3)
\end{align*}
\]

Translate to an equation.

Solve this equation.

Clear the fractions by multiplying by the LCD of all the fractions in the equation.

Simplify.

Let’s check if this works.

\[
\begin{array}{ll}
\text{Walk 3 mph (0.5 hour)} & = 1.5 \text{ miles} \\
\text{Bike 6 mph (0.25 hour)} & = 1.5 \text{ miles}
\end{array}
\]

Yes, either way Naoko travels 1.5 miles to school.

Naoko’s walking speed is 3 mph and her speed riding her bike is 6 mph.

Example \(\PageIndex{23}\)

Suzy takes 50 minutes to hike uphill from the parking lot to the lookout tower. It takes her 30 minutes to hike back down to the parking lot. Her speed going downhill is 1.2 miles per hour faster than her speed going uphill. Find Suzy’s uphill and downhill speeds.

Answer

Suzy’s speed uphill is 1.81.8 mph and downhill is three mph.
Example \(\PageIndex{24}\)

Llewyn takes 45 minutes to drive his boat upstream from the dock to his favorite fishing spot. It takes him 30 minutes to drive the boat back downstream to the dock. The boat's speed going downstream is four miles per hour faster than its speed going upstream. Find the boat's upstream and downstream speeds.

**Answer**

The boat's speed upstream is eight mph and downstream is 12 mph.

In the distance, rate and time formula, time represents the actual amount of elapsed time (in hours, minutes, etc.). If a problem gives us starting and ending times as clock times, we must find the elapsed time in order to use the formula.

Example \(\PageIndex{25}\)

Cruz is training to compete in a triathlon. He left his house at 6:00 and ran until 7:30. Then he rode his bike until 9:45. He covered a total distance of 51 miles. His speed when biking was 1.6 times his speed when running. Find Cruz’s biking and running speeds.

**Answer**

A diagram will help us model this trip.

![Diagram](image)

Next, we create a table to organize the information. We know the total distance is 51 miles. We are looking for the rate of speed for each part of the trip. The rate while biking is 1.6 times the rate of running. If we let \(r\) = the rate running, then the rate biking is \(1.6r\).

The times here are given as clock times. Cruz started from home at 6:00 a.m. and started biking at 7:30 a.m. So he spent 1.5 hours running. Then he biked from 7:30 a.m until 9:45 a.m. So he spent 2.25 hours biking.

Now, we multiply the rates by the times.

<table>
<thead>
<tr>
<th></th>
<th>Rate (mph)</th>
<th>Time (hrs)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>run</td>
<td>(r)</td>
<td>1.5</td>
<td>(1.5r)</td>
</tr>
<tr>
<td>bike</td>
<td>(1.6r)</td>
<td>2.25</td>
<td>(2.25(1.6r))</td>
</tr>
</tbody>
</table>

By looking at the diagram, we can see that the sum of the distance running and the distance biking is 255 miles.
Translate to an equation.

\[
1.5r + 2.25(1.6r) = 51
\]

Solve this equation.

\[
1.5r + 2.25(1.6r) = 51
1.5r + 3.6r = 51
5.1r = 51
r = 10 \text{ mph running}
1.6r \text{ biking speed}
1.6(10) \text{ 16 mph}
\]

Check.

\[
\begin{array}{lll}
\text{Run } 10 \text{ mph (1.5 hour)} &= &{15 \text{ mi}} \\
\text{Bike } 16 \text{ mph (2.25 hour)} &= &{36 \text{ mi}} \\
{} &{} &{} &{51 \text{ mi}} \\
\end{array}
\]

Example \((\PageIndex{26})\)

Hamilton loves to travel to Las Vegas, 255 miles from his home in Orange County. On his last trip, he left his house at 2:00 p.m. The first part of his trip was on congested city freeways. At 4:00 pm, the traffic cleared and he was able to drive through the desert at a speed 1.75 times faster than when he drove in the congested area. He arrived in Las Vegas at 6:30 p.m. How fast was he driving during each part of his trip?

Answer

Hamilton drove 40 mph in the city and 70 mph in the desert.

Example \((\PageIndex{27})\)

Phuong left home on his bicycle at 10:00. He rode on the flat street until 11:15, then rode uphill until 11:45. He rode a total of 31 miles. His speed riding uphill was 0.6 times his speed on the flat street. Find his speed biking uphill and on the flat street.

Answer

Phuong rode uphill at a speed of 12 mph and on the flat street at 20 mph.
Key Concepts

- **Total Value of Coins**
  For the same type of coin, the total value of a number of coins is found by using the model
  \[ \text{number} \times \text{value} = \text{total value} \]
  - \textit{number} is the number of coins
  - \textit{value} is the value of each coin
  - \textit{total value} is the total value of all the coins

- **How to solve coin word problems.**
  1. **Read** the problem. Make sure all the words and ideas are understood.
     Determine the types of coins involved.
     Create a table to organize the information.
     Label the columns “type,” “number,” “value,” “total value.”
     List the types of coins.
     Write in the value of each type of coin.
     Write in the total value of all the coins.

     | Type | Number | Value ($) | Total Value ($) |
     |------|--------|-----------|-----------------|
     |      |        |           |                 |
     |      |        |           |                 |
     |      |        |           |                 |

  2. **Identify** what you are looking for.

  3. **Name** what you are looking for. Choose a variable to represent that quantity.
     Use variable expressions to represent the number of each type of coin and write them in the table.
     Multiply the number times the value to get the total value of each type of coin.

  4. **Translate** into an equation.
     It may be helpful to restate the problem in one sentence with all the important information. Then, translate the sentence into an equation.
     Write the equation by adding the total values of all the types of coins.

  5. **Solve** the equation using good algebra techniques.

  6. **Check** the answer in the problem and make sure it makes sense.

  7. **Answer** the question with a complete sentence.

- **How To Solve a Uniform Motion Application**
  1. **Read** the problem. Make sure all the words and ideas are understood.
     Draw a diagram to illustrate what it happening.
     Create a table to organize the information.
     Label the columns rate, time, distance.
     List the two scenarios.
     Write in the information you know.
2. **Identify** what you are looking for.

3. **Name** what you are looking for. Choose a variable to represent that quantity.
   
   Complete the chart.
   
   Use variable expressions to represent that quantity in each row.
   
   Multiply the rate times the time to get the distance.

4. **Translate** into an equation.
   
   Restate the problem in one sentence with all the important information.
   
   Then, translate the sentence into an equation.

5. **Solve** the equation using good algebra techniques.

6. **Check** the answer in the problem and make sure it makes sense.

7. **Answer** the question with a complete sentence.