5.2.2: Changing Elevation

Lesson

Let's solve problems about adding signed numbers.

Exercise \(\PageIndex{1}\): That's the Opposite

1. Draw arrows on a number line to represent these situations:

   a. The temperature was -5 degrees. Then the temperature rose 5 degrees.

   Figure \(\PageIndex{1}\)

   b. A climber was 30 feet above sea level. Then she descended 30 feet.

   Figure \(\PageIndex{2}\)

2. What's the opposite?

   a. Running 150 feet east.
   b. Jumping down 10 steps.
   c. Pouring 8 gallons into a fish tank.

Exercise \(\PageIndex{2}\): Cliffs and Caves
Explore the applet and then answer the questions.

1. A mountaineer is climbing on a cliff. She is 200 feet above the ground. If she climbs up, this will be a positive change. If she climbs down, this will be a negative change.

![Image of a mountaineer climbing](image)

Figure \(\PageIndex{3}\): By makamuki0. Public Domain. Pixabay. Source.

a. Complete the table.

<table>
<thead>
<tr>
<th>starting elevation (feet)</th>
<th>change (feet)</th>
<th>final elevation (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(+200)</td>
<td>(+75) up</td>
</tr>
<tr>
<td>B</td>
<td>(+200)</td>
<td>(+75) down</td>
</tr>
<tr>
<td>C</td>
<td>(+200)</td>
<td>(+200) down</td>
</tr>
<tr>
<td>D</td>
<td>(+200)</td>
<td>(+25)</td>
</tr>
</tbody>
</table>

b. Write an addition equation and draw a number line diagram for B. Include the starting elevation, change, and final elevation in your diagram.

2. A spelunker is down in a cave next to the cliff. If she climbs down deeper into the cave, this will be a negative change. If
she climbs up, whether inside the cave or out of the cave and up the cliff, this will be a positive change.

a. Complete the table.

<table>
<thead>
<tr>
<th>starting elevation (feet)</th>
<th>change (feet)</th>
<th>final elevation (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(-20)</td>
<td>down (-15)</td>
</tr>
<tr>
<td>B</td>
<td>(-20)</td>
<td>up (10)</td>
</tr>
<tr>
<td>C</td>
<td>(-20)</td>
<td>up (20)</td>
</tr>
<tr>
<td>D</td>
<td>(-20)</td>
<td>up (25)</td>
</tr>
<tr>
<td>E</td>
<td>(-20)</td>
<td>down (-50)</td>
</tr>
</tbody>
</table>

b. Write an addition equation and draw a number line diagram for C and D. Include the starting elevation, change, and final elevation in your diagram.

3. What does the expression \((-45+60)\) tell us about the spelunker? What does the value of the expression tell us?

Exercise \(\PageIndex{3}\): Adding Rational Numbers

Find the sums.

1. \((-35 + (30+5))\)
2. \((-0.15 +(-0.85)+12.5)\)
3. \(\left(\frac{1}{2}\right)+\left(-\frac{3}{4}\right)\)

Are you ready for more?

Find the sum without a calculator.

\(\left(10+21+32+43+54+(-54)+(-43)+(-32)+(-21)+(-10)\right)\)

Exercise \(\PageIndex{4}\): School Supply Number Line

Your teacher will give you a long strip of paper.

Follow these instructions to create a number line.

1. Fold the paper in half along its length and along its width.
2. Unfold the paper and draw a line along each crease.
3. Label the line in the middle of the paper 0. Label the right end of the paper \(+(+1)\) and the left end of the paper \((-1)\).
4. Select two objects of different lengths, for example a pen and a gluestick. The length of the longer object is \(a\) and the length of the shorter object is \(b\).
5. Use the objects to measure and label each of the following points on your number line.
\[
\begin{array}{lllll}
& a & & 2b & & -b \\
& b & & a+b & & a-b \\
& 2a & & -a & & b-a \\
\end{array}
\]

6. Complete each statement using <, >, or =. Use your number line to explain your reasoning.
   a. \( a \) _____ \( b \)
   b. \( -a \) _____ \( -b \)
   c. \( a+b \) _____ \( b-a \)
   d. \( a+b \) _____ \( b-a \)
   e. \( a+b \) _____ \( -a+b \)

**Summary**

The opposite of a number is the same distance from 0 but on the other side of 0.

![Figure 4](image1)

Figure 4: A number line with the numbers negative 10 through 10 indicated. An arrow starts at 0, points to the left, and ends at negative 9. There is a solid dot indicated at negative 9. A second arrow starts at 0, points to the right, and ends at 9. There is a solid dot indicated at 9.

The opposite of -9 is 9. When we add opposites, we always get 0. This diagram shows that \((-9)+9=0\).

![Figure 5](image2)

Figure 5: A number line with the numbers negative 10 through 10 indicated. An arrow starts at 0, points to the right, and ends at 9. A second arrow starts at 9, points to the left, and ends at 0. There is a solid dot indicated at 0.

When we add two numbers with the same sign, the arrows that represent them point in the same direction. When we put the arrows tip to tail, we see the sum has the same sign.

![Figure 6](image3)

Figure 6: Two identical number lines with the numbers negative 10 through 10 indicated. On the top number line, an arrow starts at 0, points to the right, and ends at 4. A second arrow starts at 4, points to the right, and ends at 7. There is a solid dot indicated at 7. On the bottom number line, an arrow starts at 0, points to the left, and ends at negative 5. A second arrow starts at negative 5, points to the left, and ends at negative 9. There is a solid dot indicated at negative 9.

To find the sum, we add the magnitudes and give it the correct sign. For example, \((-5)+(-4)=-9\).

On the other hand, when we add two numbers with different signs, we subtract their magnitudes (because the arrows point in
the opposite direction) and give it the sign of the number with the larger magnitude. For example, \((-5)+12=+(12-5))

Figure \(\PageIndex{7}\): A number line with the numbers negative 10 through 10 indicated. An arrow starts at 0, points to the left, and ends at negative five. A second arrow starts at negative five, points to the right, and ends at 7. There is a solid dot indicated at 7.

Practice

Exercise \(\PageIndex{5}\)

What is the final elevation if

1. A bird starts at 20 m and changes 16 m?
2. A butterfly starts at 20 m and changes -16 m?
3. A diver starts at 5 m and changes -16 m?
4. A whale starts at -9 m and changes 11 m?
5. A fish starts at -9 meters and changes -11 meters?

Exercise \(\PageIndex{6}\)

One of the particles in an atom is called an electron. It has a charge of -1. Another particle in an atom is a proton. It has charge of +1. The charge of an atom is the sum of the charges of the electrons and the protons. A carbon atom has an overall charge of 0, because it has 6 electrons and 6 protons and \((-6+6=0))

Find the overall charge for the rest of the elements on the list.

<table>
<thead>
<tr>
<th>charge from electrons</th>
<th>charge from protons</th>
<th>overall charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>carbon</td>
<td>((-6))</td>
<td>((+6))</td>
</tr>
<tr>
<td>neon</td>
<td>((-10))</td>
<td>((+10))</td>
</tr>
<tr>
<td>oxide</td>
<td>((-10))</td>
<td>((+8))</td>
</tr>
<tr>
<td>copper</td>
<td>((-27))</td>
<td>((+29))</td>
</tr>
<tr>
<td>tin</td>
<td>((-50))</td>
<td>((+50))</td>
</tr>
</tbody>
</table>

Exercise \(\PageIndex{7}\)

Add.

1. \(14.7+28.9\)
2. \((-9.2+4.4)\)
3. \((-81.4+(-12))\)
4. \((51.8+(-0.8))\)

Exercise \(\PageIndex{8}\)

Last week, the price, in dollars, of a gallon of gasoline was \(g\). This week, the price of gasoline per gallon increased by 5%. Which expressions represent this week's price, in dollars, of a gallon of gasoline? Select all that apply.

A. \((g+0.05)\)
B. \((g+0.05g)\)
C. \((1.05g)\)
D. \((0.05g)\)
E. \((1+0.05)g\)

(From Unit 4.2.3)

Exercise \(\PageIndex{9}\)

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

1. Annie's Attic is giving away $5 off coupons.
   Table \(\PageIndex{4}\)
<table>
<thead>
<tr>
<th>original price</th>
<th>sale price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15</td>
<td>$10</td>
</tr>
<tr>
<td>$25</td>
<td>$20</td>
</tr>
<tr>
<td>$35</td>
<td>$30</td>
</tr>
</tbody>
</table>

2. Bettie's Boutique is having a 20% off sale.
   Table \(\PageIndex{5}\)
<table>
<thead>
<tr>
<th>original price</th>
<th>sale price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15</td>
<td>$12</td>
</tr>
<tr>
<td>$25</td>
<td>$20</td>
</tr>
<tr>
<td>$35</td>
<td>$28</td>
</tr>
</tbody>
</table>

(From Unit 2.3.1)