5.1: VAT 8512

Obv.

1 A triangle. 30 the width. In the inside two plots,

2 the upper surface over the lower surface, (7')went beyond.

3 The lower descendant over the upper descendant, 20 went beyond.

4 The descendants and the bar what?

5 And the surfaces of the two plots what?

6 You, 30 the width posit, (7') which the upper surface over the lower surface went beyond posit,

7 and 20 which the lower descendant over the upper descendant went beyond posit.

8 igi 20 which the lower descendant over the upper descendant went beyond

9 detach: (3' (prime)) to (7') which the upper surface over the lower surface went beyond

10 raise, 21 may your head hold!

11 21 to 30 the width join: 51

12 together with 51 make hold: (43'21)
13 21 which your head holds together with 21

14 make hold: (7°21') to (43°21') join: (50°42').

15 (50°42') to two break: (25°21').


17 From 39, 21 the made-hold tear out, 18.

18 18 which you have left is the bar.

19 Well, if 18 is the bar,

20 the descendants and the surfaces of the two plots what?

21 You, 21 which together with itself you have made hold, from 51

22 tear out: 30 you leave. 30 which you have left

23 to two break, 15 to 30 which you have left raise,

24 (7°30') may your head hold!

Edge

1 18 the bar together with 18 make hold:

2 (5°24') from (7°30') which your head holds

3 tear out: (2°6') you leave.

Rev.

1 What to (2°6') may I posit

2 which \((7º\) which the upper surface over the lower surface went beyond gives me?

3 \((3^\circ20^\prime\)\) posit. \((3^\circ2^\prime\)\) to \((2°6')\) raise, \((7º\)\) it gives you.

4 30 the width over 18 the bar what goes beyond? 12 it goes beyond.

5 12 to \((3^\circ20^\prime\)\) which you have posited raise, 40.
Many Old Babylonian mathematical problems deal with the partition of fields. The mathematical substance may vary—sometimes the shape of the field is irrelevant and only the area is given together with the specific conditions for its division; sometimes, as here, what is asked for is a division of a particular geometric shape.

Already before 2200 bce, Mesopotamian surveyors knew how to divide a trapezium into two equal parts by means of a parallel transversal; we shall return in a short while to how they did it. A similar division of a triangle cannot be made exactly without the use of irrational numbers—which means that it could not be done by the Old Babylonian calculators (except with approximation, which was not among the normal teaching aims).

The present problem deals with a variant of the triangle division which can be performed exactly. As we see in lines Obv. 1–3 and as shown in Figure 5.1, a triangular field is divided into two parcels (an “upper surface” and a “lower surface”) by a “bar,” that is, a parallel transversal. For simplicity we may assume the triangle to be rectangular. It is almost certain that the author of the text did as much, and that the “descendants” are thus part of the side; but if we interpret the “descendants” as heights, the calculations are valid for an oblique triangle too.
The two parcels are thus unequal in area. However, we know the difference between their areas, as well as the difference between the appurtenant "descendants." The solution makes use of an unsuspected and elegant ruse and may therefore be difficult to follow.

Lines Obv. 8-10 "raise" the igi of the difference between the two "descendants" to the difference between the two "surfaces." This means that the text finds the width of a rectangle whose length corresponds to the difference between the partial heights and whose area equals the difference between the partial areas. This width (which is 21) is first memorized and then "joined" to the width of the triangle.

The outcome is a triangle with an attached rectangle—all in all the trapezium shown in Figure 5.1. When prolonging the bar, producing a parallel transversal of the trapezium, we discover that it divides the trapezium into two equal parts—and that is the problem the surveyors had known to solve for half a millennium or more.

Lines Obv. 11-16 shows how they had done it: the square on the bisecting transversal is determined as the average between the squares of the parallel sides. The operations that are used ("making hold" and "breaking") show that the process is really thought in terms of geometric squares and average. Figure (5.2) shows why the procedure leads to the correct result. By definition, the average is equidistant from the two extremes. Therefore the gnomon between 21 and 39 must equal that between 39 and 51 \( (39^2 - 21^2 = 51^2 - 39^2) \); half of these gnomons—the two parts of the shaded trapezium—must therefore also be equal. In the first instance this only concerns a trapezium cut out along the diagonal of a square, but we may imagine the square drawn long (into a rectangle) and perhaps twisted into a parallelogram; none of these operations changes the ratio between ares or parallel linear extensions, and they allow the creation of an arbitrary trapezium. This trapezium will still be bisected, and the sum of the squares on the parallel sides will be still be twice that of the parallel transversal.
We may take note that the operation of "drawing long" is the same as the change of scale in one direction which we have encountered in the solution of non-normalized problems, and which was also used in TMS XIII, the oil trade (see page 70); we shall meet it again in a moment in the present problem.

Possibly the rule was first found on the basis of concentric squares (Figure 5.3)—the geometric configuration represented by two or several concentrically nested squares was much appreciated in Babylonian mathematics and may have been so already in the third millennium (as it remained popular among master builders until the Renaissance); the principle of the argument evidently remains the same.
Figure 5.3: The bisection of the trapezium explained by concentric squares.

Line Obv. 17 thus finds the bisecting transversal; it turns out to be \((39)\), and the "bar" between the two original parcels must therefore be \((39-21=18)\).

The next steps may seem strange. Lines Obv. 21-22 appear to calculate the width of the triangle, but this was one of the given magnitudes of the problem. this means no doubt that we have effectively left behind Figure 5.1, and that the argument is now based on something like Figure 5.2. When we eliminate the additional width 21 we are left with a triangle that corresponds to the initial triangle but which is isosceles—Figure 5.4.
In order to find the "upper descendant" the text makes the false position that the shortened and isosceles triangle is the one we are looking for. Its length (the sum of the "descendants") is then equal to the width, that is, to \(30\). In order to find the true triangle we will have to change the scale in the direction of the length.

Lines Obv. 23-24 calculate that the area of the false triangle is \(7 \cdot 30\). The two areas in white are equal, and their sum must be \((2 \cdot \text{cdot} \left(\frac{1}{2} \cdot 18 \cdot 18\right)) = 30\). The shaded area—which corresponds to the difference between the two parcels—must therefore be \((7 \cdot 30 - 2 \cdot 6)\) (edge 1-3).

But we know that the difference is \(7\) and not \(2\). Lines Rev. 1-3 therefore establish that the difference \(2\) that results from the false position must be multiplied by \(3\) if we are to find the true difference \(7\). Since the width is already what it should be, it is the length and the "descendants" that must be multiplied by this factor. The "upper descendant" will thus be \((3 \cdot 18 \cdot 18) = 40\) (line Rev. 6). Afterwards everything is quite simple; it could have been even simpler, but the road that is chosen agrees better with the pedagogical style which we know for example from TMS XVI #1, and it is probably more fruitful from a didactical point of view.

The way this problem is solved certainly differs from what we have encountered so far. But there are also common features that become more conspicuous in a bird's eye view.

The change of scale in one direction we already know as an algebraic technique. A no less conspicuous difference—the absence of a quadratic completion, that is, of the “Akkadian method”—points to another family characteristic: the introduction of an auxiliary figure that is first "joined" and then "torn out."

Less evident but fundamental is the "analytic" character of the methods. Since Greek antiquity, the solution of a mathematical problem is called "analytic" if it starts from the presupposition that the problem is already solved; that allows us to examine—to analyze—the characteristics of the solution in order to understand how to construct it.\(^1\)

A solution by equation is always analytical. In order to understand that we may look again at our modern solution of TMS XIII, the oil trade (page 70). According to the starting hypothesis, the quantity of sìla that is bought for 1 shekel of silver is a known number, and we call it \(a\). We do the same with the sales rate (which we call \(v\)). The total investment is hence \(M \div a\), the total sales price \(M \div v\), and the profit therefore \(w = \text{frac}\{M\} - \text{frac}\{M\} \cdot a\) \(v\). Then we multiply by \(v \times a\), and so forth.

That is, we treat \(a\) and \(v\) as if they were known numbers; we pretend to have a solution and we describe its characteristics. Afterwards we derive the consequences—and find in the end that \(a=11\), \(v=7\).

Even the Old Babylonian cut-and-paste solutions are analytic. Presupposing that we know a solution to the oil problem we express it as rectangle of area \((12 \cdot 50\)) of which a part of length \((40\) corresponds to the oil profit. Then we examine the characteristics of this solution, and find the normalization factor by which we should multiply in order to get a difference 4 between the sides, and so on.

The solution to the present problem is also analytical. We presuppose that the triangle has been completed by a rectangle in
such a way that the prolonged "bar" divides resulting trapezium in equal parts, after which we calculate how much the width of the rectangle must be if that shall be the case; and so on. Even though it has its justification, the distinction between "algebra" (Problems that are easily translated into modern equations) and "quasi-algebra" seems less important in the perspective of the Old Babylonian texts than in ours.