4.1: Euclidean geometry

Euclidean geometry, sometimes called parabolic geometry, is a geometry that follows a set of propositions that are based on Euclid's five postulates.

There are two types of Euclidean geometry: plane geometry, which is two-dimensional Euclidean geometry, and solid geometry, which is three-dimensional Euclidean geometry.

The most basic terms of geometry are a point, a line, and a plane. A point has no dimension (length or width), but it does have a location. A line is straight and extends infinitely in the opposite directions. A plane is a flat surface that extends indefinitely.

Points

Definitions

Please refer to the image below for examples.

Collinear points: points that lie on the same straight line or line segment. Points A, B, and C are collinear.
**Line Segment**: a straight line with two endpoints. Lines AC, EF, and GH are line segments.

**Ray**: a part of a straight line that contains a specific point. Any of the below line segments could be considered a ray.

**Intersection point**: the point where two straight lines intersect, or cross. The point I is the intersection point for lines EF and GH.

**Midpoint**: a point in the exact middle of a given straight line segment. Point B is the midpoint of line AC.

**THINKING OUT LOUD**

How many different lines can you draw through a fixed point? How many different lines can you draw through two fixed points? How many different lines can you draw through three fixed points?

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**Angles**

**Definitions**

**Angle**: \(\measuredangle ACB\). Normally, Angle is measured in degrees (\(^0\)) or in radians rad).

**Right angle**: Angles which measure 90° - \(\measuredangle ABC\)

**Obtuse angle**: Angles which measure > 90° - \(\measuredangle CDE\)

**Acute angle**: Angles which measure < 90° - \(\measuredangle FDE\)

**Straight angle**: Angles which measure 180° \(\measuredangle CDF\)

**Reflex angle**: A reflex angle is an angle, that is measured > 180°, which adds to an angle to make 360° - \(\measuredangle CDE\)'s reflex angle is \(\measuredangle CDF + \measuredangle FDE\)
Adjacent angles: Have the same vertex and share a side. \( \measuredangle HRL, \measuredangle HRO \) are adjacent.

Complementary angles: add up to 90°. \( \measuredangle PRQ, \measuredangle QRI \) are complementary angles.

Supplementary angles: add up to 180°. \( \measuredangle JSN, \measuredangle NSK \) are supplementary angles.

Vertical angles (X property): Angles which share line segments and vertexes are equivalent. \( \measuredangle JSR, \measuredangle OST \) are vertical angles. They share the same degree value.

Corresponding angles (F property): Angles which share a line segment that intersects with parallel lines, and are in the same relative position on each respective parallel line, are equivalent. \( \measuredangle IRQ, \measuredangle KUQ \) are corresponding angles. They share the same degree value.

Alternate interior angles (Z property): Angles which share a line segment that intersects with parallel lines, and are in opposite relative positions on each respective parallel line, are equivalent. \( \measuredangle HRS, \measuredangle RST \) are alternate interior angles. They share the same degree value.

Bisecting an Angle: To bisect an angle is to draw a line concurrent line through the angle's vertex which splits the angle exactly in half. This is possible using a compass and an unmarked straight-edge.

Trisecting an Angle: To trisect an angle is to use the same procedure as bisecting an angle, but to use two lines and split the angle exactly in thirds. This is an ancient impossibility - it is impossible to accomplish using a compass and an unmarked straight-edge.
Lines

Definitions

Parallel lines: Lines which, drawn on a 2-dimensional plane, may extend forever in either direction without ever intersecting. Lines $\langle HI \rangle$ and $\langle JK \rangle$ are parallel.

Perpendicular lines: Lines which intersect at exactly a 90° angle. Lines $\langle HI \rangle$ and $\langle MP \rangle$ are perpendicular.

Concurrent lines: Lines which all intersect at the same point. Lines $\langle HI, LQ, MP, NO \rangle$ are concurrent.

Skew lines: Lines which, drawn in a 3-dimensional space, are both neither parallel nor perpendicular and do not intersect.

Perpendicular Bisector: A line that is perpendicular to a given line and bisects it is called perpendicular bisector.

Example $(\PageIndex{1})$:

Draw a cube and connect all the edges. Can you find two skew lines?

Planes

Definition

A plane is a two-dimensional space that extends infinitely in all directions. For example, the graph of functions takes place on a Cartesian plane or a plane with coordinates. The plane continues in both the x and, y directions.

The points, lines, and planes are objects with the relations given by following axioms:

1. There is a unique line passing through two distinct points.
2. If two points lie in a plane, then any plane containing those points lies in the plane.
3. There is a unique plane containing three non-collinear points.
4. If two planes meet, then their intersection is a line.

Euclid's five postulates

Euclid's five postulates can be stated as follows

1. It is possible to draw a straight line segment joining any two points.
2. It is possible to indefinitely extend any straight line segment continuously in a straight line.
3. Given any straight line segment, it is possible to draw a circle having the segment as a radius and one endpoint as its center.
4. All right angles are equal to each other or congruent.
5. Through a given point not on a given straight line, only one line can be drawn parallel to a given line.