4.5: Multiply and Divide Mixed Numbers and Complex Fractions (Part 1)

Skills to Develop

- Multiply and divide mixed numbers
- Translate phrases to expressions with fractions
- Simplify complex fractions
- Simplify expressions written with a fraction bar

be prepared!

Before you get started, take this readiness quiz.

1. Divide and reduce, if possible: \((4 + 5) ÷ (10 − 7)\). If you missed this problem, review Example 3.2.8.
2. Multiply and write the answer in simplified form: \(\dfrac{1}{8} \cdot \dfrac{2}{3}\). If you missed this problem, review Example 4.2.7.
3. Convert \(2 \dfrac{3}{5}\) into an improper fraction. If you missed this problem, review Example 4.1.11.

Multiply and Divide Mixed Numbers

In the previous section, you learned how to multiply and divide fractions. All of the examples there used either proper or improper fractions. What happens when you are asked to multiply or divide mixed numbers? Remember that we can convert a mixed number to an improper fraction. And you learned how to do that in Visualize Fractions.

Example \(\langle\PageIndex{1}\rangle\): multiply
Multiply: \(3 \dfrac{1}{3} \cdot \dfrac{5}{8}\)

**Solution**

Convert \(3 \dfrac{1}{3}\) to an improper fraction. \(\dfrac{10}{3} \cdot \dfrac{5}{8}\)

Multiply. \(\dfrac{10 \cdot 5}{3 \cdot 8}\)

Look for common factors. \(\dfrac{\cancel{2} \cdot 5 \cdot 5}{3 \cdot \cancel{2} \cdot 4}\)

Remove common factors. \(\dfrac{5 \cdot 5}{3 \cdot 4}\)

Simplify. \(\dfrac{25}{12}\)

Notice that we left the answer as an improper fraction, \(\dfrac{25}{12}\), and did not convert it to a mixed number. In algebra, it is preferable to write answers as improper fractions instead of mixed numbers. This avoids any possible confusion between \(\dfrac{2}{12}\) and \(\dfrac{2 \cdot 1}{12}\).

Exercise \((\PageIndex{1})\)

Multiply, and write your answer in simplified form: \(5 \dfrac{2}{3} \cdot \dfrac{6}{17}\).

**Answer**

\(2\)

Exercise \((\PageIndex{2})\)

Multiply, and write your answer in simplified form: \(\dfrac{3}{7} \cdot 5 \dfrac{1}{4}\).

**Answer**

\(\dfrac{9}{4}\)

**HOW TO: MULTIPLY OR DIVIDE MIXED NUMBERS**

**Step 1.** Convert the mixed numbers to improper fractions.

**Step 2.** Follow the rules for fraction multiplication or division.

**Step 3.** Simplify if possible.

**Example \((\PageIndex{2})\):**

Multiply, and write your answer in simplified form: \(2 \dfrac{4}{5} \left(− 1 \dfrac{7}{8}\right)\).

**Solution**
Convert mixed numbers to improper fractions.

Multiply. \( \dfrac{14}{5} \times (-1 \dfrac{7}{8}) \)

Look for common factors. \(- \dfrac{\cancel{2} \cdot 7 \cdot \cancel{5} \cdot 3}{\cancel{5} \cdot \cancel{2} \cdot 4}\)

Remove common factors. \(- \dfrac{7 \cdot 3}{4}\)

Simplify. \(- \dfrac{21}{4}\)

Exercise \(\PageIndex{3}\)

Multiply, and write your answer in simplified form. \(5 \dfrac{5}{7} \times (-2 \dfrac{5}{8})\).

Answer

\(-15\)

Exercise \(\PageIndex{4}\)

Multiply, and write your answer in simplified form. \(-3 \dfrac{2}{5} \times 4 \dfrac{1}{6}\).

Answer

\(-\dfrac{85}{6}\)

Example \(\PageIndex{3}\): divide

Divide, and write your answer in simplified form: \(3 \dfrac{4}{7} \div 5\).

Solution

Convert mixed numbers to improper fractions. \(\dfrac{25}{7} \div \dfrac{5}{1}\)

Multiply the first fraction by the reciprocal of the second. \(\dfrac{25}{7} \times \dfrac{1}{5}\)

Multiply. \(\dfrac{25 \times 1}{7 \times 5}\)

Look for common factors. \(\dfrac{\cancel{5} \cdot 5 \cdot 1}{7 \cdot \cancel{5}}\)

Remove common factors. \(\dfrac{5 \cdot 1}{7}\)

Simplify. \(\dfrac{5}{7}\)

Exercise \(\PageIndex{5}\)
Divide, and write your answer in simplified form: $4 \dfrac{3}{8} ÷ 7$.

**Answer**

$\dfrac{5}{8}$

Exercise \(\PageIndex{6}\)

Divide, and write your answer in simplified form: $2 \dfrac{5}{8} ÷ 3$.

**Answer**

$\dfrac{7}{8}$

Example \(\PageIndex{4}\): divide

Divide: $2 \dfrac{1}{2} ÷ 1 \dfrac{1}{4}$.

**Solution**

Convert mixed numbers to improper fractions.

$\dfrac{5}{2} ÷ \dfrac{5}{4}$

Multiply the first fraction by the reciprocal of the second.

$\dfrac{5}{2} \cdot \dfrac{4}{5}$

Multiply.

$\dfrac{5 \cdot 4}{2 \cdot 5}$

Look for common factors.

$\dfrac{\cancel{5} \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot 1 \cdot \cancel{5}}$

Remove common factors.

$\dfrac{2}{1}$

Simplify.

$2$

Exercise \(\PageIndex{7}\)

Divide, and write your answer in simplified form: $2 \dfrac{2}{3} ÷ 1 \dfrac{1}{3}$.

**Answer**

$2$

Exercise \(\PageIndex{8}\)

Divide, and write your answer in simplified form: $3 \dfrac{3}{4} ÷ 1 \dfrac{1}{2}$.
Translate Phrases to Expressions with Fractions

The words *quotient* and *ratio* are often used to describe fractions. In *Subtract Whole Numbers*, we defined quotient as the result of division. The quotient of \( a \) and \( b \) is the result you get from dividing \( a \) by \( b \), or \( \frac{a}{b} \). Let’s practice translating some phrases into algebraic expressions using these terms.

Example \( \PageIndex{5} \): translate

Translate the phrase into an algebraic expression: “the quotient of \( 3x \) and \( 8 \).”

**Solution**

The keyword is *quotient*; it tells us that the operation is division. Look for the words *of* and *and* to find the numbers to divide.

The quotient of \( 3x \) and \( 8 \).

This tells us that we need to divide \( 3x \) by \( 8 \). \( \frac{3x}{8} \)

Exercise \( \PageIndex{9} \)

Translate the phrase into an algebraic expression: the quotient of \( 9s \) and \( 14 \).

**Answer**

\( \frac{9s}{14} \)

Exercise \( \PageIndex{10} \)

Translate the phrase into an algebraic expression: the quotient of \( 5y \) and \( 6 \).

**Answer**

\( \frac{5y}{6} \)

Example \( \PageIndex{6} \):

Translate the phrase into an algebraic expression: the quotient of the difference of \( m \) and \( n \), and \( p \).

**Solution**

We are looking for the *quotient* of the *difference* of \( m \) and \( n \), and \( p \). This means we want to divide the difference of
\( \frac{m}{n} \) and \( \frac{n}{p} \) by \( \frac{p}{n} \).

\[\frac{m - n}{p}\] 

**Exercise \( \PageIndex{11} \)**

Translate the phrase into an algebraic expression: the quotient of the difference of \( a \) and \( b \), and \( cd \).

**Answer**

\[\frac{a - b}{cd}\]

**Exercise \( \PageIndex{12} \)**

Translate the phrase into an algebraic expression: the quotient of the sum of \( p \) and \( q \), and \( r \).

**Answer**

\[\frac{p + q}{r}\]

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**Simplify Complex Fractions**

Our work with fractions so far has included proper fractions, improper fractions, and mixed numbers. Another kind of fraction is called **complex fraction**, which is a fraction in which the numerator or the denominator contains a fraction. Some examples of complex fractions are:

\[\frac{\frac{6}{7}}{3} \quad \frac{\frac{3}{4}}{\frac{5}{8}} \quad \frac{\frac{x}{2}}{\frac{5}{6}}\]

To simplify a complex fraction, remember that the fraction bar means division. So the complex fraction \(\frac{\frac{3}{4}}{\frac{5}{8}}\) can be written as \(\frac{3}{4} \div \frac{5}{8}\).

**Example \( \PageIndex{7} \): simplify**

Simplify: \(\frac{\frac{3}{4}}{\frac{5}{8}}\).

**Solution**

1. **Rewrite as division.**
   \[\frac{3}{4} \div \frac{5}{8}\]
2. **Multiply the first fraction by the reciprocal of the second.**
   \[\frac{3}{4} \cdot \frac{8}{5}\]
3. **Multiply.**
   \[\frac{3 \cdot 8}{4 \cdot 5}\]
4. **Look for common factors.**
   \[\frac{3 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 5}\]

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Remove common factors and simplify. \(\dfrac{6}{5}\)

Exercise \(\PageIndex{13}\)

Simplify: \(\dfrac{\dfrac{2}{3}}{\dfrac{5}{6}}\).

\textbf{Answer}

\(\dfrac{4}{5}\)

Exercise \(\PageIndex{14}\)

Simplify: \(\dfrac{\dfrac{3}{7}}{\dfrac{6}{11}}\).

\textbf{Answer}

\(\dfrac{11}{14}\)

\textbf{HOW TO: SIMPLIFY A COMPLEX FRACTION}

\textbf{Step 1.} Rewrite the complex fraction as a division problem.

\textbf{Step 2.} Follow the rules for dividing fractions.

\textbf{Step 3.} Simplify if possible.

\textbf{Example \(\PageIndex{8}\): simplify}

Simplify: \(- \dfrac{\dfrac{6}{7}}{3}\).

\textbf{Solution}

\textbf{Rewrite as division.}

\(- \dfrac{6}{7} \div 3\)

Multiply the first fraction by the reciprocal of the second.

\(- \dfrac{6}{7} \cdot \dfrac{1}{3}\)

Multiply; the product will be negative.

\(- \dfrac{6 \cdot 1}{7 \cdot 3}\)

Look for common factors.

\(- \dfrac{\cancel{3} \cdot 2 \cdot 1}{7 \cdot \cancel{3}}\)

Remove common factors and simplify.

\(- \dfrac{2}{7}\)

Exercise \(\PageIndex{15}\)

Simplify: \(- \dfrac{\dfrac{8}{7}}{4}\).
Exercise \(\PageIndex{16}\)
Simplify: \(− \dfrac{3}{\dfrac{9}{10}}\).

Answer
\(-\dfrac{10}{3}\)

Example \(\PageIndex{9}\): simplify
Simplify: \(\dfrac{\dfrac{x}{2}}{\dfrac{xy}{6}}\).

Solution

Rewrite as division. \(\dfrac{x}{2} \div \dfrac{xy}{6}\)
Multiply the first fraction by the reciprocal of the second. \(\dfrac{x}{2} \cdot \dfrac{6}{xy}\)
Multiply. \(\dfrac{x \cdot 6}{2 \cdot xy}\)
Look for common factors. \(\dfrac{\cancel{x} \cdot 3 \cdot \cancel{2}}{\cancel{2} \cdot \cancel{x} \cdot y}\)
Remove common factors and simplify. \(\dfrac{3}{y}\)

Exercise \(\PageIndex{17}\)
Simplify: \(\dfrac{\dfrac{a}{8}}{\dfrac{ab}{6}}\).

Answer
\(\dfrac{3}{4b}\)

Exercise \(\PageIndex{18}\)
Simplify: \(\dfrac{\dfrac{p}{2}}{\dfrac{pq}{8}}\).

Answer
\(\dfrac{4}{q}\)

Example \(\PageIndex{10}\): simplify
Simplify: \( \dfrac{2 \dfrac{3}{4}}{\dfrac{1}{8}} \).

**Solution**

Rewrite as division. \( 2 \dfrac{3}{4} \div \dfrac{1}{8} \)

Change the mixed number to an improper fraction. \( \dfrac{11}{4} \div \dfrac{1}{8} \)

Multiply the first fraction by the reciprocal of the second. \( \dfrac{11}{4} \cdot \dfrac{8}{1} \)

Multiply. \( \dfrac{11 \cdot 8}{4 \cdot 1} \)

Look for common factors. \( \dfrac{11 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 1} \)

Remove common factors and simplify. \( 22 \)

**Exercise** \( \PageIndex{19} \)

Simplify: \( \dfrac{\dfrac{5}{7}}{1 \dfrac{2}{5}} \).

**Answer** \( \dfrac{25}{49} \)

**Exercise** \( \PageIndex{20} \)

Simplify: \( \dfrac{\dfrac{8}{5}}{3 \dfrac{1}{5}} \).

**Answer** \( \dfrac{1}{2} \)

**Contributors and Attributions**

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