7.5: Properties of Identity, Inverses, and Zero

Skills to Develop
- Recognize the identity properties of addition and multiplication
- Use the inverse properties of addition and multiplication
- Use the properties of zero
- Simplify expressions using the properties of identities, inverses, and zero

be prepared!

Before you get started, take this readiness quiz.

1. Find the opposite of \(-4\). If you missed this problem, review Example 3.1.3.
2. Find the reciprocal of \(\frac{5}{2}\). If you missed this problem, review Example 4.4.11.
3. Multiply: \(\frac{3a}{5} \cdot \frac{9}{2a}\). If you missed this problem, review Example 4.3.9.

Recognize the Identity Properties of Addition and Multiplication

What happens when we add zero to any number? Adding zero doesn’t change the value. For this reason, we call 0 the **additive identity**. For example,

\[
\begin{align*}
13 + 0 & = 13 \\
0 + (-3x) & = -3x
\end{align*}
\]

Definition: Identity Properties
The identity property of addition: for any real number \(a\),

\[
\begin{align*}
[a + 0 &= a \\
0 + a &= a]
\end{align*}
\]

0 is called the additive identity

The identity property of multiplication: for any real number \(a\)

\[
\begin{align*}
[a \cdot 1 &= a \\
1 \cdot a &= a]
\end{align*}
\]

1 is called the multiplicative identity

Example \(\PageIndex{1}\):

Identify whether each equation demonstrates the identity property of addition or multiplication. (a) \(7 + 0 = 7\) (b) \(-16(1) = −16\)

Solution

(a) \(7 + 0 = 7\)

We are adding 0. We are using the identity property of addition.

(b) \(-16(1) = −16\)

We are multiplying by 1. We are using the identity property of multiplication.

Exercise \(\PageIndex{1}\):

Identify whether each equation demonstrates the identity property of addition or multiplication: (a) \(23 + 0 = 23\) (b) \(-37(1) = −37\).

Answer a

identity property of addition

Answer b

identity property of multiplication

Exercise \(\PageIndex{2}\):

Identify whether each equation demonstrates the identity property of addition or multiplication: (a) \(1 \cdot 29 = 29\) (b) \(14 + 0 = 14\).
Answer a

identity property of multiplication

Answer b

additive identity

Use the Inverse Properties of Addition and Multiplication

What number added to 5 gives the additive identity, 0?

\[ 5 + _____ = 0 \quad \text{We know } (5 + (\textcolor{red}{-5}) = 0) \]

What number added to \(-6\) gives the additive identity, 0?

\[ -6 + _____ = 0 \quad \text{We know } (-6 + (\textcolor{red}{6}) = 0) \]

Notice that in each case, the missing number was the opposite of the number. We call \(-a\) the additive inverse of \(a\). The opposite of a number is its additive inverse. A number and its opposite add to 0, which is the additive identity.

What number multiplied by \(\frac{2}{3}\) gives the multiplicative identity, 1? In other words, two-thirds times what results in 1?

\[ \frac{2}{3} \cdot _____ = 1 \quad \text{We know } \left(\frac{2}{3}\right) \cdot \textcolor{red}{\frac{3}{2}} = 1 \]

What number multiplied by 2 gives the multiplicative identity, 1? In other words, two times what results in 1?

\[ 2 \cdot _____ = 1 \quad \text{We know } 2 \cdot \textcolor{red}{\frac{1}{2}} = 1 \]

Notice that in each case, the missing number was the reciprocal of the number.

We call \(\frac{1}{a}\) the multiplicative inverse of \(a\) \((a \neq 0)\). The reciprocal of a number is its multiplicative inverse. A number and its reciprocal multiply to 1, which is the multiplicative identity.

We’ll formally state the Inverse Properties here:

Definition: Inverse Properties

Inverse Property of Addition for any real number \(a\),

\[ [a + (-a) = 0] \]
\( -a \) is the additive inverse of \( a \).

Inverse Property of Multiplication for any real number \( a \neq 0 \),

\[
\frac{a}{a} \cdot \frac{1}{a} = 1
\]

\( \frac{1}{a} \) is the multiplicative inverse of \( a \).

Example \( \PageIndex{2} \):

Find the additive inverse of each expression: (a) 13 (b) \(-\frac{5}{8}\) (c) 0.6.

Solution

To find the additive inverse, we find the opposite.

a. The additive inverse of 13 is its opposite, \(-13\).

b. The additive inverse of \(-\frac{5}{8}\) is its opposite, \(\frac{5}{8}\).

c. The additive inverse of 0.6 is its opposite, \(-0.6\).

Exercise \( \PageIndex{3} \):

Find the additive inverse of each expression: (a) 18 (b) \(\frac{7}{9}\) (c) 1.2.

Answer a

\(-18\)

Answer b

\(-\frac{7}{9}\)

Answer c

\(-1.2\)

Exercise \( \PageIndex{4} \):

Find the additive inverse of each expression: (a) 47 (b) \(\frac{7}{13}\) (c) 8.4.

Answer a

\(-47\)
Example \(\PageIndex{3}\):

Find the multiplicative inverse: (a) 9 (b) \(- \dfrac{1}{9}\) (c) 0.9.

\textbf{Solution}

To find the additive inverse, we find the opposite.

a. The multiplicative inverse of 9 is its reciprocal, \(\dfrac{1}{9}\).

b. The multiplicative inverse of \(- \dfrac{1}{9}\) is its reciprocal, -9.

c. To find the multiplicative inverse of 0.9, we first convert 0.9 to a fraction, \(\dfrac{9}{10}\). Then we find the reciprocal, \(\dfrac{10}{9}\).

Exercise \(\PageIndex{5}\):

Find the multiplicative inverse: (a) 5 (b) \(- \dfrac{1}{7}\) (c) 0.3.

\textbf{Answer a}

\(\dfrac{1}{5}\)

\textbf{Answer b}

\(-7\)

\textbf{Answer c}

\(\dfrac{10}{3}\)

Exercise \(\PageIndex{6}\):

Find the multiplicative inverse: (a) 18 (b) \(- \dfrac{4}{5}\) (c) 0.6.

\textbf{Answer a}

\(\dfrac{1}{18}\)
Use the Properties of Zero

We have already learned that zero is the additive identity, since it can be added to any number without changing the number’s identity. But zero also has some special properties when it comes to multiplication and division.

Multiplication by Zero

What happens when you multiply a number by 0? Multiplying by 0 makes the product equal zero. The product of any real number and 0 is 0.

Definition: Multiplication by Zero

For any real number a,

\[ a \cdot 0 = 0 \quad \text{and} \quad 0 \cdot a = 0 \]

Example:

Simplify: (a) \(-8 \cdot 0\) (b) \(\frac{5}{12} \cdot 0\) (c) \(0(2.94)\).

Solution

(a) \(-8 \cdot 0\)

The product of any real number and 0 is 0. 0

(b) \(\frac{5}{12} \cdot 0\)

The product of any real number and 0 is 0. 0

(c) \(0(2.94)\)

The product of any real number and 0 is 0. 0
Exercise \(\PageIndex{7}\):

Simplify: (a) \(-14 \cdot 0\) (b) \(0 \cdot \frac{2}{3}\) (c) \((16.5) \cdot 0\).

Answer a

\(0\)

Answer b

\(0\)

Answer c

\(0\)

Exercise \(\PageIndex{8}\):

Simplify: (a) \((1.95) \cdot 0\) (b) \(0 \cdot (-17)\) (c) \(0 \cdot \frac{5}{4}\).

Answer a

\(0\)

Answer b

\(0\)

Answer c

\(0\)

---

**Dividing with Zero**

What about dividing with 0? Think about a real example: if there are no cookies in the cookie jar and three people want to share them, how many cookies would each person get? There are 0 cookies to share, so each person gets 0 cookies.

\([0 \div 3 = 0]\)

Remember that we can always check division with the related multiplication fact. So, we know that

\([0 \div 3 = 0; \text{ because}; 0 \cdot 3 = 0 \cdot \text{dotp}]\)

Definition: Division of Zero
For any real number \(a\), except 0, \(\frac{0}{a}\) = 0 and \(0 \div a = 0\).

Zero divided by any real number except zero is zero.

Example \(\PageIndex{5}\):

Simplify: (a) \(0 \div 5\) (b) \(\frac{0}{-2}\) (c) \(0 \div \frac{7}{8}\).

Solution

(a) \(0 \div 5\)

Zero divided by any real number, except 0, is zero. \(0\)

(b) \(\frac{0}{-2}\)

Zero divided by any real number, except 0, is zero. \(0\)

(c) \(0 \div \frac{7}{8}\)

Zero divided by any real number, except 0, is zero. \(0\)

Exercise \(\PageIndex{9}\):

Simplify: (a) \(0 \div 11\) (b) \(\frac{0}{-6}\) (c) \(0 \div \frac{3}{10}\).

Answer a

\(0\)

Answer b

\(0\)

Answer c

\(0\)

Exercise \(\PageIndex{10}\):

Simplify: (a) \(0 \div \frac{8}{3}\) (b) \(0 \div (-10)\) (c) \(0 \div 12.75\).

Answer a

\(0\)
Now let’s think about dividing a number by zero. What is the result of dividing 4 by 0? Think about the related multiplication fact. Is there a number that multiplied by 0 gives 4?

\[ 4 \div 0 = \_\_\_ \text{ means } \_\_\_ \times 0 = 4 \]

Since any real number multiplied by 0 equals 0, there is no real number that can be multiplied by 0 to obtain 4. We can conclude that there is no answer to \( 4 \div 0 \), and so we say that division by zero is undefined.

**Definition: Division by Zero**

For any real number \( a \), \( \frac{a}{0} \), and \( a \div 0 \) are undefined.

Division by zero is undefined.

**Example** 

Simplify: (a) \( 7.5 \div 0 \) (b) \( \frac{-32}{0} \) (c) \( \frac{4}{9} \div 0 \).

**Solution**

(a) \( 7.5 \div 0 \)

Division by zero is undefined. undefined

(b) \( \frac{-32}{0} \)

Division by zero is undefined. undefined

(c) \( \frac{4}{9} \div 0 \)

Division by zero is undefined. undefined

**Exercise**

Simplify: (a) \( 16.4 \div 0 \) (b) \( \frac{-2}{0} \) (c) \( \frac{1}{5} \div 0 \).
Exercise \(\PageIndex{12}\):

Simplify: (a) \(\dfrac{-5}{0}\) (b) \(96.9 \div 0\) (c) \(\dfrac{4}{15}\) \(\div 0\).

**Answer a**

undefined

**Answer b**

undefined

**Answer c**

undefined

We summarize the properties of zero.

**Definition: properties of zero**

**Multiplication by Zero:** For any real number \(a\),

\[a \cdot 0 = 0 \quad 0 \cdot a = 0\]

The product of any number and 0 is 0.

**Division by Zero:** For any real number \(a, a \neq 0\)

\[\dfrac{0}{a} = 0\]

Zero divided by any real number, except itself, is zero.
Simplify Expressions using the Properties of Identities, Inverses, and Zero

We will now practice using the properties of identities, inverses, and zero to simplify expressions.

Example \( \PageIndex{7} \):
Simplify: \( 3x + 15 − 3x \).

**Solution**

Notice the additive inverses, \( 3x \) and \( −3x \).
\[ 0 + 15 \]
Add.
\[ 15 \]

Exercise \( \PageIndex{13} \):
Simplify: \( −12z + 9 + 12z \).

**Answer**

\[ 9 \]

Exercise \( \PageIndex{14} \):
Simplify: \( −25u − 18 + 25u \).

**Answer**

\[ -18 \]

Example \( \PageIndex{8} \):
Simplify: \( 4(0.25q) \).

**Solution**

Regroup, using the associative property.
\[ [4(0.25)]q \]
Multiply.
\[ 1.00q \]
Simplify; 1 is the multiplicative identity.
\[ q \]

Exercise \( \PageIndex{15} \):
Simplify: \(2(0.5p)\).

**Answer**

\[p\]

Exercise \(\PageIndex{16}\):

Simplify: \(25(0.04r)\).

**Answer**

\[r\]

Example \(\PageIndex{9}\):

Simplify: \(\dfrac{0}{n + 5}\), where \(n \neq -5\).

**Solution**

Zero divided by any real number except itself is zero.

\[0\]

Exercise \(\PageIndex{17}\):

Simplify: \(\dfrac{0}{m + 7}\), where \(m \neq -7\).

**Answer**

\[0\]

Exercise \(\PageIndex{18}\):

Simplify: \(\dfrac{0}{d - 4}\), where \(d \neq 4\).

**Answer**

\[0\]

Example \(\PageIndex{10}\):

Simplify: \(\dfrac{10 - 3p}{0}\).

**Solution**

Division by zero is undefined.

\[\text{undefined}\]
Exercise \(\PageIndex{19}\): 

Simplify: \(\dfrac{18 - 6c}{0}\).

**Answer**

undefined

Exercise \(\PageIndex{20}\):

Simplify: \(\dfrac{15 - 4q}{0}\).

**Answer**

undefined

Example \(\PageIndex{11}\):

Simplify: \(\dfrac{3}{4} \cdot \dfrac{4}{3}(6x + 12)\).

**Solution**

We cannot combine the terms in parentheses, so we multiply the two fractions first.

Multiply; the product of reciprocals is 1.

\[1(6x + 12)\]

Simplify by recognizing the multiplicative identity.

\[6x + 12\]

Exercise \(\PageIndex{21}\):

Simplify: \(\dfrac{2}{5} \cdot \dfrac{5}{2}(20y + 50)\).

**Answer**

\[20y + 50\]

Exercise \(\PageIndex{22}\):

Simplify: \(\dfrac{3}{8} \cdot \dfrac{8}{3}(12z + 16)\).

**Answer**

\[12z + 16\]

All the properties of real numbers we have used in this chapter are summarized in Table \(\PageIndex{1}\).
### Table: Properties of Real Numbers

<table>
<thead>
<tr>
<th>Property</th>
<th>Of Addition</th>
<th>Of Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Commutative Property</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>If a and b are real numbers then…</td>
<td>a + b = b + a</td>
<td>a • b = b • a</td>
</tr>
<tr>
<td><strong>Associative Property</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>If a, b, and c are real numbers then…</td>
<td>(a + b) + c = a + (b + c)</td>
<td>(a • b) • c = (b • c)</td>
</tr>
<tr>
<td><strong>Identity Property</strong></td>
<td>0 is the additive identity</td>
<td>1 is the multiplicative identity</td>
</tr>
<tr>
<td>For any real number a,</td>
<td>a + 0 = a</td>
<td>a • 1 = a</td>
</tr>
<tr>
<td></td>
<td>0 + a = a</td>
<td>1 • a = a</td>
</tr>
<tr>
<td><strong>Inverse Property</strong></td>
<td>−a is the additive inverse of a</td>
<td>a, a ≠ 0</td>
</tr>
<tr>
<td>For any real number a,</td>
<td>a + (−a) = 0</td>
<td>a • 1/a = 1</td>
</tr>
<tr>
<td><strong>Distributive Property</strong></td>
<td>If a, b, c are real numbers, then a(b + c) = ab + ac</td>
<td></td>
</tr>
<tr>
<td><strong>Properties of Zero</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For any real number a,</td>
<td>a • 0 = 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 • a = 0</td>
<td></td>
</tr>
<tr>
<td>For any real number a where a ≠ 0</td>
<td>(\dfrac{a}{0} = 0)</td>
<td>(\dfrac{0}{a}) is undefined</td>
</tr>
</tbody>
</table>

ACCESS ADDITIONAL ONLINE RESOURCES

**Multiplying and Dividing Involving Zero**

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Practice Makes Perfect

Recognize the Identity Properties of Addition and Multiplication

In the following exercises, identify whether each example is using the identity property of addition or multiplication.

158. $101 + 0 = 101$
159. $\frac{3}{5}(1) = \frac{3}{5}$
160. $-9 \cdot 1 = -9$
161. $0 + 64 = 64$

Use the Inverse Properties of Addition and Multiplication

In the following exercises, find the multiplicative inverse.

162. $8$
163. $14$
164. $-17$
165. $-19$
166. $\frac{7}{12}$
167. $\frac{8}{13}$
168. $-\frac{3}{10}$
169. $-\frac{5}{12}$
170. $0.8$
171. $0.4$
172. $-0.2$
173. $-0.5$

Use the Properties of Zero

In the following exercises, simplify using the properties of zero.

174. $48 \cdot 0$
175. $\frac{0}{6}$
176. $\frac{0}{3}$
177. $22 \cdot 0$
178. $0 + \frac{11}{12}$
179. $\frac{6}{0}$
Simplify Expressions using the Properties of Identities, Inverses, and Zero

In the following exercises, simplify using the properties of identities, inverses, and zero.

186. \[19a + 44 - 19a\]
187. \[27c + 16 - 27c\]
188. \[38 + 11r - 38\]
189. \[92 + 31s - 92\]
190. \[10(0.1d)\]
191. \[100(0.01p)\]
192. \[5(0.6q)\]
193. \[40(0.05n)\]
194. \[\dfrac{0}{r + 20}\], where \(r \neq -20\)
195. \[\dfrac{0}{s + 13}\], where \(s \neq -13\)
196. \[\dfrac{0}{u - 4.99}\], where \(u \neq 4.99\)
197. \[\dfrac{0}{v - 65.1}\], where \(v \neq 65.1\)
198. \[0 + (x - \dfrac{1}{2})\], where \(x \neq \dfrac{1}{2}\)
199. \[0 + (y - \dfrac{1}{6})\], where \(y \neq \dfrac{1}{6}\)
200. \[\dfrac{32 - 5a}{0}\], where \(32 - 5a \neq 0\)
201. \[\dfrac{28 - 9b}{0}\], where \(28 - 9b \neq 0\)
202. \[\dfrac{2.1 + 0.4c}{0}\], where \(2.1 + 0.4c \neq 0\)
203. \[\dfrac{1.75 + 9 f}{0}\], where \(1.75 + 9 f \neq 0\)
204. \[\left(\dfrac{3}{4} + \dfrac{9}{10}m \right) / 0\], where \(\dfrac{3}{4} + \dfrac{9}{10}m \neq 0\)
205. \[\left(\dfrac{5}{16}n - \dfrac{3}{7}\right) / 0\], where \(\dfrac{5}{16}n - \dfrac{3}{7} \neq 0\)
206. \[\dfrac{9}{10} \cdot \dfrac{10}{9}(18p - 21)\]
207. \[\dfrac{5}{7} \cdot \dfrac{7}{5}(20q - 35)\]
208. \[15 \cdot \dfrac{5}{6}(4d + 10)\]
209. \[18 \cdot \dfrac{5}{6}(15h + 24)\]
Everyday Math

210. **Insurance copayment** Carrie had to have 5 fillings done. Each filling cost $80. Her dental insurance required her to pay 20% of the cost. Calculate Carrie's cost
   a. by finding her copay for each filling, then finding her total cost for 5 fillings, and
   b. by multiplying $5(0.20)(80)$.
   c. Which of the Properties of Real Numbers did you use for part (b)?

211. **Cooking time** Helen bought a 24-pound turkey for her family's Thanksgiving dinner and wants to know what time to put the turkey in the oven. She wants to allow 20 minutes per pound cooking time.
   a. Calculate the length of time needed to roast the turkey by multiplying $24 \cdot 20$ to find the number of minutes and then multiplying the product by $\frac{1}{60}$ to convert minutes into hours.
   b. Multiply $24\left(20 \cdot \frac{1}{60}\right)$.
   c. Which of the Properties of Real Numbers allows you to multiply $24\left(20 \cdot \frac{1}{60}\right)$ instead of $(24 \cdot 20)\left(\frac{1}{60}\right)$?

Writing Exercises

212. In your own words, describe the difference between the additive inverse and the multiplicative inverse of a number.

213. How can the use of the properties of real numbers make it easier to simplify expressions?

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

<table>
<thead>
<tr>
<th>I can...</th>
<th>Confidently</th>
<th>With some help</th>
<th>No-I don’t get it!</th>
</tr>
</thead>
<tbody>
<tr>
<td>recognize the Identity Properties of Addition and Multiplication</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>use the Inverse Properties of Addition and Multiplication</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>simplify expressions using the properties of identities, inverses, and zero</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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