6.1: Extracting Square Roots and Completing the Square

Learning Objectives

- Solve certain quadratic equations by extracting square roots.
- Solve any quadratic equation by completing the square.

Extracting Square Roots

Recall that a quadratic equation is in **standard form** if it is equal to 0:

\[ a x^2 + b x + c = 0 \]

where \(a, b,\) and \(c\) are real numbers and \(a \neq 0\). A solution to such an equation is a root of the quadratic function defined by \(f(x) = ax^2 + bx + c\). Quadratic equations can have two real solutions, one real solution, or no real solution—in which case there will be two complex solutions. If the quadratic expression factors, then we can solve the equation by factoring. For example, we can solve \((4x^2 - 9 = 0)\) by factoring as follows:

\[
\begin{aligned}
4x^2 - 9 &= 0 \\
(2x + 3)(2x - 3) &= 0 \\
2x + 3 &= 0 \quad \text{or} \quad 2x - 3 &= 0 \\
2x &= -3 \quad \text{or} \quad 2x &= 3 \\
x &= -\frac{3}{2} \quad \text{or} \quad x &= \frac{3}{2}
\end{aligned}
\]

The two solutions are \(\pm \frac{3}{2}\). Here we use \(\pm\) to write the two solutions in a more compact form. The goal in this section is to develop an alternative method that can be used to easily solve equations where \(b = 0\), giving the form
The equation \((4x^2 - 9 = 0)\) is in this form and can be solved by first isolating \((x^2)\).

\[
\begin{align*}
4x^2 - 9 &= 0 \\
4x^2 &= 9 \\
x^2 &= \frac{9}{4}
\end{align*}
\]

If we take the square root of both sides of this equation, we obtain the following:

\[
\begin{align*}
\sqrt{x^2} &= \sqrt{\frac{9}{4}} \\
|x| &= \frac{3}{2}
\end{align*}
\]

Here we see that \((x = \pm \frac{3}{2})\) are solutions to the resulting equation. In general, this describes the square root property; for any real number \((k)\),

\[
\text{If } x^2 = k, \text{ then } x = \pm \sqrt{k}
\]

Applying the square root property as a means of solving a quadratic equation is called extracting the root. This method allows us to solve equations that do not factor.

Example \((\text{PageIndex}1)\):

Solve: \((9x^2 - 8 = 0)\).

**Solution**

Notice that the quadratic expression on the left does not factor. However, it is in the form \((ax^2 + c = 0)\) and so we can solve it by extracting the roots. Begin by isolating \((x^2)\).

\[
\begin{align*}
9x^2 - 8 &= 0 \\
9x^2 &= 8 \\
x^2 &= \frac{8}{9}
\end{align*}
\]

Next, apply the square root property. Remember to include the \((\pm)\) and simplify.

\[
\begin{align*}
x &= \pm \sqrt{\frac{8}{9}} \\
&= \pm \frac{2\sqrt{2}}{3}
\end{align*}
\]

For completeness, check that these two real solutions solve the original quadratic equation.

**Table \((\text{PageIndex}1)\)**

<table>
<thead>
<tr>
<th>Check (x = -\frac{2\sqrt{2}}{3})</th>
<th>Check (x = \frac{2\sqrt{2}}{3})</th>
</tr>
</thead>
</table>
| \begin{align*}
9x^2 - 8 &= 0 \\
9\left(-\frac{2\sqrt{2}}{3}\right)^2 - 8 &= 0 \\
&= 0 \\
\end{align*} | \begin{align*}
9x^2 - 8 &= 0 \\
9\left(\frac{2\sqrt{2}}{3}\right)^2 - 8 &= 0 \\
&= 0 \\
\end{align*} |

**Answer:**
Two real solutions, $\pm \frac{2\sqrt{2}}{3}$

Sometimes quadratic equations have no real solution. In this case, the solutions will be complex numbers.

Example \(\PageIndex{2}\):

Solve: \((x^2+25=0)\).

**Solution**

Begin by isolating \((x^2)\) and then apply the square root property.

\[
\begin{aligned}
x^2 + 25 &= 0 \\
x^2 &= -25 \\
x &= \pm \sqrt{-25}
\end{aligned}
\]

After applying the square root property, we are left with the square root of a negative number. Therefore, there is no real solution to this equation; the solutions are complex. We can write these solutions in terms of the imaginary unit \(i = \sqrt{-1}\).

\[
\begin{aligned}
x &= \pm \sqrt{-25} \\
&= \pm \sqrt{-1 \cdot 25} \\
&= \pm i \cdot 5 \\
&= \pm 5i
\end{aligned}
\]

Table \(\PageIndex{2}\)

<table>
<thead>
<tr>
<th>Check ((x=-5i))</th>
<th>Check ((x=5i))</th>
</tr>
</thead>
</table>
| \(\begin{aligned}
\text{Check } (x=-5i) \\
&x^2 + 25 = 0 \\
&(-5i)^2 + 25 = 0 \\
&25i^2 + 25 = 0 \\
&25(-1) + 25 = 0 \\
&-25 + 25 = 0 \color{red}{\checkmark}
\end{aligned}\) | \(\begin{aligned}
\text{Check } (x=5i) \\
&x^2 + 25 = 0 \\
&(5i)^2 + 25 = 0 \\
&25i^2 + 25 = 0 \\
&25(-1) + 25 = 0 \\
&-25 + 25 = 0 \color{red}{\checkmark}
\end{aligned}\) |

**Answer:**

Two complex solutions, \(\pm 5i\).

Exercise \(\PageIndex{1}\)

Solve: \((2x^2+3=0)\).

**Answer**

The solutions are \(\pm \frac{\sqrt{6}}{2}\).

www.youtube.com/v/9ff7QGhFytQ

Consider solving the following equation:

\(((x+5)^2 = 9)\)

To solve this equation by factoring, first square \((x+5)\) and then put the equation in standard form, equal to zero, by

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subtracting \(9\) from both sides.

\[
\begin{aligned}
(x + 5)^2 \quad & \quad \{2\} \quad & \quad = \quad 9 \\
x^2 + 10x + 25 \quad & \quad = \quad 9 \\
x^2 + 10x + 16 \quad & \quad = \quad 0
\end{aligned}
\]

Factor and then apply the zero-product property.

\[
\begin{aligned}
x^2 + 10x + 16 \quad & \quad = \quad 0 \\
x^2 + 8x + 2 \quad & \quad = \quad 0 \\
x + 8 \quad & \quad = \quad 0 \\
x \quad & \quad = \quad -8 \\
x + 2 \quad & \quad = \quad 0 \\
x \quad & \quad = \quad -2
\end{aligned}
\]

The two solutions are \((-8)\) and \((-2)\). When an equation is in this form, we can obtain the solutions in fewer steps by extracting the roots.

Example \(\PageIndex{3}\):

Solve by extracting roots: \(((x + 5)^2 \quad \{2\} \quad = \quad 9)\).

**Solution**

The term with the square factor is isolated so we begin by applying the square root property.

\[
\begin{aligned}
(x + 5)^2 \quad & \quad = \quad 9 \quad \color{Cerulean}{Apply\:the\:square\:root\:property.} \\
x + 5 \quad & \quad = \quad \pm \sqrt{9} \quad \color{Cerulean}{Simplify.} \\
x \quad & \quad = \quad -5 \pm 3
\end{aligned}
\]

At this point, separate the “plus or minus” into two equations and solve each individually.

\[
\begin{array}{l}
x = -5 + 3 \quad \text{or} \quad x = -5 - 3 \\
x = -2 \quad \quad \quad x = -8
\end{array}
\]

**Answer:**

The solutions are \((-2)\) and \((-8)\).

In addition to fewer steps, this method allows us to solve equations that do not factor.

Example \(\PageIndex{4}\):

Solve: \((2(x - 2)^2 \quad \{2\} \quad - \quad 5 \quad = \quad 0)\).

**Solution**

Begin by isolating the term with the square factor.

\[
\begin{aligned}
2(x - 2)^2 \quad & \quad = \quad 5 \\
2(x - 2)^2 \quad & \quad = \quad \frac{5}{2}
\end{aligned}
\]

Next, extract the roots, solve for \((x)\), and then simplify.
\( x - 2 = \pm \sqrt{\frac{5}{2}} \)
\[
\text{Rationalize the denominator.}
\]
\[
\begin{array}{c}
\text{Answer:}
\end{array}
\]
\[
\text{The solutions are } \frac{4 - \sqrt{10}}{2} \text{ and } \frac{4 + \sqrt{10}}{2}.
\]

Exercise \(\PageIndex{2}\)

Solve: \((2 \cdot 3x - 1)^2 + 9 = 0\).
\[
\text{Answer}
\]
\[
\text{The solutions are } \frac{1}{3} \pm \frac{\sqrt{2}}{2}i.
\]

Completing the Square

In this section, we will devise a method for rewriting any quadratic equation of the form

\[(ax^2 + bx + c = 0)\]

as an equation of the form

\[(x - p)^2 = q\]

This process is called **completing the square**. As we have seen, quadratic equations in this form can be easily solved by extracting roots. We begin by examining perfect square trinomials:

\[
\begin{array}{c}
(x + 3)^2 = x^2 + 6x + 9
\end{array}
\]

The last term, 9, is the square of one-half of the coefficient of \(x\). In general, this is true for any perfect square trinomial of the form \(x^2 + bx + c\).

\[
\begin{array}{c}
\text{In other words, any trinomial of the form } (x^2 + bx + c) \text{ will be a perfect square trinomial if }
\end{array}
\]

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\(c = \left( \frac{b}{2} \right)^2\)

**Note**

It is important to point out that the leading coefficient must be equal to 1 for this to be true.

**Example (PageIndex{5})**:

Complete the square: \(x^2 - 6x + \text{?} = (x + \text{?})^2\).

**Solution**

In this example, the coefficient \(b\) of the middle term is \(-6\). Find the value that completes the square as follows:

\[
\left( \frac{b}{2} \right)^2 = \left( \frac{-6}{2} \right)^2 = (-3)^2 = 9
\]

The value that completes the square is 9.

\[
\begin{aligned}
x^2 - 6x + 9 & = (x - 3)(x - 3) \\
& = (x - 3)^2
\end{aligned}
\]

**Answer**:

\(x^2 - 6x + 9 = (x - 3)^2\)

**Example (PageIndex{6})**:

Complete the square: \(x^2 + x + \text{?} = (x + \text{?})^2\).

**Solution**

Here \(b=1\). Find the value that will complete the square as follows:

\[
\left( \frac{b}{2} \right)^2 = \left( \frac{1}{2} \right)^2 = \frac{1}{4}
\]

The value \(\frac{1}{4}\) completes the square:

\[
\begin{aligned}
x^2 + x + \frac{1}{4} & = \left( x + \frac{1}{2} \right)^2
\end{aligned}
\]

**Answer**:

\(x^2 + x + \frac{1}{4} = \left( x + \frac{1}{2} \right)^2\)
We can use this technique to solve quadratic equations. The idea is to take any quadratic equation in standard form and complete the square so that we can solve it by extracting roots. The following are general steps for solving a quadratic equation with leading coefficient 1 in standard form by completing the square.

Example \(\PageIndex{7}\):

Solve by completing the square: \((x^2 - 8x - 2 = 0)\).

**Solution**

It is important to notice that the leading coefficient is \((1)\).

**Step 1**: Add or subtract the constant term to obtain an equation of the form \((x^2 + bx = c)\). Here we add \(2\) to both sides of the equation.

\[
\begin{aligned}
x^2 - 8x - 2 &= 0 \\
x^2 - 8x &= 2
\end{aligned}
\]

**Step 2**: Use \((\frac{b}{2})^2\) to determine the value that completes the square. In this case, \((b = -8)\).

\[
\begin{aligned}
\left(\frac{b}{2}\right)^2 &= \left(\frac{-8}{2}\right)^2 = (-4)^2 = \text{Cerulean}{16}\end{aligned}
\]

**Step 3**: Add \((\frac{b}{2})^2\) to both sides of the equation and complete the square.

\[
\begin{aligned}
x^2 - 8x & = 2 \\
x^2 - 8x + 16 & = 2 + 16 \\
(x - 4)^2 & = 18
\end{aligned}
\]

**Step 4**: Solve by extracting roots.

\[
\begin{aligned}
(x - 4)^2 & = 18 \\
x - 4 & = \pm \sqrt{18} \\
x & = 4 \pm 3\sqrt{2}
\end{aligned}
\]

**Answer**:

The solutions are \((4 - 3\sqrt{2})\) and \((4 + 3\sqrt{2})\). The check is left to the reader.

Example \(\PageIndex{8}\):

Solve by completing the square: \((x^2 + 2x - 48 = 0)\).

**Solution**

Begin by adding \((48)\) to both sides

\[
\begin{aligned}
x^2 + 2x - 48 & = 0 \\
x^2 + 2x & = 48
\end{aligned}
\]

Next, find the value that completes the square using \((b = 2)\).
To complete the square, add \( (1) \) to both sides, complete the square, and then solve by extracting the roots.

\[
\begin{aligned}
(x + 1)^2 &= 49 \\
x + 1 &= \pm \sqrt{49} \\
x &= -1 \pm 7
\end{aligned}
\]

At this point, separate the “plus or minus” into two equations and solve each individually.

\[
\begin{array}{l}
x = -8 \\
x = 6
\end{array}
\]

**Answer:**

The solutions are \(-8\) and \(6\).

**Note**

In the previous example the solutions are integers. If this is the case, then the original equation will factor.

\[
\begin{array}{c}
x^2 + 2x - 48 = 0 \\
(x - 6)(x + 8) = 0
\end{array}
\]

If an equation factors, we can solve it by factoring. However, not all quadratic equations will factor. Furthermore, equations often have complex solutions.

**Example \( \PageIndex{9} \):**

Solve by completing the square: \(x^2 - 10x + 26 = 0\).

**Solution**

Begin by subtracting \(26\) from both sides of the equation.

\[
\begin{aligned}
x^2 - 10x + 26 &= 0 \\
x^2 - 10x &= -26
\end{aligned}
\]

Here \(b=-10\), and we determine the value that completes the square as follows:

\[
\begin{aligned}
\left(\frac{b}{2}\right)^2 &= \left(\frac{-10}{2}\right)^2 \\
&= (-5)^2 = 25
\end{aligned}
\]

To complete the square, add \(25\) to both sides of the equation.

\[
\begin{aligned}
x^2 - 10x &+ 25 = -26 + 25 \\
x^2 - 10x + 25 &= -1
\end{aligned}
\]

Factor and then solve by extracting roots.
\[
\begin{aligned}
&x^2 - 10x + 25 = -1 \\
&(x - 5)(x - 5) = -1 \\
&(x - 5)^2 = -1 \\
&x - 5 = \pm \sqrt{-1} \\
&x = 5 \pm i
\end{aligned}
\]

Answer:

The solutions are \((5 \pm i)\).

Exercise \((PageIndex{3})\)

Solve by completing the square: \((x^2 - 2x - 17 = 0)\).

Answer

The solutions are \((x = 1 \pm 3\sqrt{2})\).

www.youtube.com/v/i8WVWpe-Ct0

The coefficient of \((x)\) is not always divisible by \((2)\).

Example \((PageIndex{10})\):

Solve by completing the square: \((x^2 + 3x+4=0)\).

Solution

Begin by subtracting \((4)\) from both sides.

\[
\begin{aligned}
&x^2 + 3x = -4
\end{aligned}
\]

Use \((b=3)\) to find the value that completes the square:

\[
\left( \frac{b}{2} \right)^2 = \left( \frac{3}{2} \right)^2 = \frac{9}{4}
\]

To complete the square, add \((\frac{9}{4})\) to both sides of the equation.

\[
\begin{aligned}
&x^2 + 3x + \frac{9}{4} = -4 + \frac{9}{4} \\
&\left( x + \frac{3}{2} \right)^2 = -\frac{7}{4}
\end{aligned}
\]

Solve by extracting roots.

\[
\begin{aligned}
&x + \frac{3}{2} = \pm \sqrt{-\frac{7}{4}} \\
&x = -\frac{3}{2} \pm \frac{\sqrt{7}}{2}i
\end{aligned}
\]

Answer:

\[
\begin{aligned}
x &= 5 \pm i
\end{aligned}
\]
The solutions are \(- \frac{3}{2} \pm \frac{\sqrt{7}}{2} i\).

So far, all of the examples have had a leading coefficient of 1. The formula \(\left( \frac{b}{2} \right)^2\) determines the value that completes the square only if the leading coefficient is 1. If this is not the case, then simply divide both sides by the leading coefficient before beginning the steps outlined for completing the square.

Example \(\PageIndex{11}\):

Solve by completing the square: \((2x^2 + 5x - 1 = 0)\).

**Solution**

Notice that the leading coefficient is 2. Therefore, divide both sides by 2 before beginning the steps required to solve by completing the square.

\[
\begin{array}{c}
\frac{2x^2 + 5x - 1}{2} = \frac{0}{2} \\
\frac{2x^2}{2} + \frac{5x}{2} - \frac{1}{2} = 0 \\
x^2 + \frac{5}{2}x - \frac{1}{2} = 0
\end{array}
\]

Add \(\frac{1}{2}\) to both sides of the equation.

\[
\begin{aligned}
x^2 + \frac{5}{2}x &= \frac{1}{2} \\
x^2 + \frac{5}{2}x + \frac{25}{16} &= \frac{1}{2} + \frac{25}{16} \Rightarrow (x + \frac{5}{4})^2 &= \frac{33}{16}
\end{aligned}
\]

Next, solve by extracting roots.

\[
\begin{aligned}
x + \frac{5}{4} &= \pm \sqrt{\frac{33}{16}} \Rightarrow x &= -\frac{5}{4} \pm \frac{\sqrt{33}}{4}
\end{aligned}
\]

**Answer:**
The solutions are $\left\{ -\frac{5}{4} \pm \frac{\sqrt{33}}{4} \right\}$.

Exercise $\PageIndex{4}$

Solve by completing the square: $3x^2 - 2x + 1 = 0$.

Answer

The solutions are $x = \left\{ \frac{1}{3} \pm \frac{\sqrt{2}}{3} i \right\}$.

www.youtube.com/v/A-i6LkqNmtY

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**Key Takeaways**

- Solve equations of the form $ax^2 + c = 0$ by extracting the roots.
- Extracting roots involves isolating the square and then applying the square root property. Remember to include "$\pm$" when taking the square root of both sides.
- After applying the square root property, solve each of the resulting equations. Be sure to simplify all radical expressions and rationalize the denominator if necessary.
- Solve any quadratic equation by completing the square.
- You can apply the square root property to solve an equation if you can first convert the equation to the form $(x - p)^2 = q$.
- To complete the square, first make sure the equation is in the form $x^2 + bx = c$. The leading coefficient must be 1. Then add the value of $(\frac{b}{2})^2$ to both sides and factor.
- The process for completing the square always works, but it may lead to some tedious calculations with fractions. This is the case when the middle term, $b$, is not divisible by 2.

Exercise $\PageIndex{5}$

Solve by factoring and then solve by extracting roots. Check answers.

1. $(x^2 - 16 = 0)$
2. $(x^2 - 36 = 0)$
3. $(9y^2 - 1 = 0)$
4. $(4y^2 - 25 = 0)$
5. $((x - 2)^2 - 1 = 0)$
6. $((x + 1)^2 - 4 = 0)$
7. $(4(y - 2)^2 - 9 = 0)$
8. $(9(y + 1)^2 - 4 = 0)$
9. $((u - 5)^2 - 25 = 0)$
10. $((u + 2)^2 - 4 = 0)$
Answer

1. \((-4,4)\)

3. \(-\frac{1}{3}, \frac{1}{3}\)

5. \((1, 3)\)

7. \((\frac{1}{2}, \frac{7}{2})\)

9. \((0,10)\)

Exercise \(\PageIndex{6}\)

Solve by extracting the roots.

1. \(x^2 = 81\)
2. \(x^2 = 1\)
3. \(y^2 = \frac{1}{9}\)
4. \(y^2 = \frac{1}{16}\)
5. \(x^2 = 12\)
6. \(x^2 = 18\)
7. \(16x^2 = 9\)
8. \(4x^2 = 25\)
9. \(2t^2 = 1\)
10. \(3t^2 = 2\)
11. \(x^2 - 40 = 0\)
12. \(x^2 - 24 = 0\)
13. \(x^2 + 1 = 0\)
14. \(x^2 + 100 = 0\)
15. \(5x^2 - 1 = 0\)
16. \(6x^2 - 5 = 0\)
17. \(8x^2 + 1 = 0\)
18. \(12x^2 + 5 = 0\)
19. \(y^2 + 4 = 0\)
20. \(y^2 + 1 = 0\)
21. \(x^2 - \frac{4}{9} = 0\)
22. \(x^2 - \frac{9}{25} = 0\)
23. \(x^2 - 8 = 0\)
24. \(t^2 - 18 = 0\)
25. \(x^2 + 8 = 0\)
26. \(x^2 + 125 = 0\)
27. \(5y^2 - 2 = 0\)
28. \(3x^2 - 1 = 0\)
29. \((x + 7)^2 - 4 = 0\)
30. \((x + 9)^2 - 36 = 0\)
31. \((x - 5)^2 - 20 = 0\)
32. \((x + 1)^2 - 28 = 0\)
33. \((3t + 2)^2 + 6 = 0\)
34. \((3t - 5)^2 + 10 = 0\)
35. \((4(3x + 1))^2 - 27 = 0\)
36. \((9(2x - 3))^2 - 8 = 0\)
37. \((2(3x - 1))^2 + 3 = 0\)
38. \((5(2x - 1))^2 + 2 = 0\)
39. \(3\left(y - \frac{2}{3}\right)^2 - \frac{3}{2} = 0\)
40. \(2\left(3y - \frac{1}{3}\right)^2 - \frac{5}{2} = 0\)
41. \(-3(t - 1)^2 + 12 = 0\)
42. \(-2(t + 1)^2 + 8 = 0\)
43. Solve for \(x : px^2 - q = 0, p, q > 0\)
44. Solve for \(x : (x - p)^2 - q = 0, p, q > 0\)
45. The diagonal of a square measures \(3\) centimeters. Find the length of each side.
46. The length of a rectangle is twice its width. If the diagonal of the rectangle measures \(10\) meters, then find the dimensions of the rectangle.
47. If a circle has an area of \(50\) square centimeters, then find its radius.
48. If a square has an area of \(27\) square centimeters, then find the length of each side.
49. The height in feet of an object dropped from an \(18\)-foot stepladder is given by \(h(t) = -16t^2 + 18\), where \(t\) represents the time in seconds after the object is dropped. How long does it take the object to hit the ground? (Hint: The height is \(0\) when the object hits the ground. Round to the nearest hundredth of a second.)
50. The height in feet of an object dropped from a \(50\)-foot platform is given by \(h(t) = -16t^2 + 50\), where \(t\) represents the time in seconds after the object is dropped. How long does it take the object to hit the ground? (Round to the nearest hundredth of a second.)
51. How high does a \(22\)-foot ladder reach if its base is \(6\) feet from the building on which it leans? Round to the nearest tenth of a foot.
52. The height of a triangle is \(\frac{1}{2}\) the length of its base. If the area of the triangle is \(72\) square meters, find the exact length of the triangle’s base.

Answer

1. \(\pm 9\)
3. \(\pm \frac{1}{3}\)
5. \(\pm 2 \sqrt{3}\)

7. \(\pm \frac{3}{4}\)

9. \(\pm \frac{\sqrt{2}}{2}\)

11. \(\pm 2 \sqrt{10}\)

13. \(\pm i\)

15. \(\pm \frac{\sqrt{5}}{5}\)

17. \(\pm \frac{\sqrt{2}}{4}i\)

19. \(\pm 2i\)

21. \(\pm \frac{2}{3}\)

23. \(\pm 2 \sqrt{2}\)

25. \(\pm 2i \sqrt{2}\)

27. \(\pm \frac{\sqrt{10}}{5}\)

29. \(-9,-5\)

31. \(5 \pm 2 \sqrt{5}\)

33. \(-\frac{2}{3} \pm \frac{\sqrt{6}}{3}i\)

35. \(\frac{-2 \pm 3 \sqrt{3}}{6}\)

37. \(\frac{1}{3} \pm \frac{\sqrt{6}}{6}i\)

39. \(\frac{4 \pm 3 \sqrt{2}}{6}\)

41. \((-1,-3)\)

43. \(x = \pm \frac{\sqrt{pq}}{p}\)

45. \(\frac{3 \sqrt{2}}{2}\) centimeters

47. \(5 \sqrt{2}\) centimeters

49. \((1.06)\) seconds
Exercise \(\PageIndex{7}\)

Complete the square.

1. \((x^2 - 2x + ? = (x - ?)^2)\)
2. \((x^2 - 4x + ? = (x - ?)^2)\)
3. \((x^2 + 10x + ? = (x + ?)^2)\)
4. \((x^2 + 12x + ? = (x + ?)^2)\)
5. \((x^2 + 7x + ? = (x + ?)^2)\)
6. \((x^2 + 5x + ? = (x + ?)^2)\)
7. \((x^2 - x + ? = (x - ?)^2)\)
8. \((x^2 - \frac{1}{2}x + ? = (x - ?)^2)\)
9. \((x^2 + \frac{2}{3}x + ? = (x + ?)^2)\)
10. \((x^2 + \frac{4}{5}x + ? = (x + ?)^2)\)

Answer

1. \((x^2 - 2x + 1 = (x - 1)^2)\)
2. \((x^2 - 4x + 4 = (x - 2)^2)\)
3. \((x^2 + 10x + 25 = (x + 5)^2)\)
4. \((x^2 + 12x + 36 = (x + 6)^2)\)
5. \((x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2)\)
6. \((x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2)\)
7. \((x^2 - x + \frac{1}{4} = \left(x - \frac{1}{2}\right)^2)\)
8. \((x^2 - \frac{1}{2}x + \frac{1}{16} = \left(x - \frac{1}{4}\right)^2)\)
9. \((x^2 + \frac{2}{3}x + \frac{1}{9} = \left(x + \frac{1}{3}\right)^2)\)

Exercise \(\PageIndex{8}\)

Solve by factoring and then solve by completing the square. Check answers.

1. \((x^2 + 2x - 8 = 0)\)
2. \((x^2 - 8x + 15 = 0)\)
3. \((y^2 + 2y - 24 = 0)\)
4. \((y^2 - 12y + 11 = 0)\)
5. \((t^2 + 3t - 28 = 0)\)
6. \((t^2 - 7t + 10 = 0)\)
7. \((2x^2 + 2x - 2 = 0)\)
8. \((3x^2 + 2x - 2 = 0)\)
9. \((2y^2 - y - 1 = 0)\)
10. \(2y^2 + 7y - 4 = 0\)

**Answer**

1. \((-4,2)\)

3. \((-6,4)\)

5. \((-7,4)\)

7. \((-\frac{1}{2}, \frac{1}{2})\)

9. \((- \frac{1}{2}, 1)\)

**Exercise**

Solve by completing the square.

1. \(x^2 + 6x - 1 = 0\)
2. \(x^2 + 8x + 10 = 0\)
3. \(x^2 - 2x - 7 = 0\)
4. \(x^2 - 6x - 3 = 0\)
5. \(y^2 - 2y + 4 = 0\)
6. \(y^2 - 4y + 9 = 0\)
7. \(t^2 + 10t - 75 = 0\)
8. \(t^2 + 12t - 108 = 0\)
9. \(u^2 - \frac{2}{3}u - \frac{1}{3} = 0\)
10. \(u^2 - \frac{4}{5}u - \frac{1}{5} = 0\)
11. \(x^2 + x - 1 = 0\)
12. \(x^2 + x - 3 = 0\)
13. \(y^2 + 3y - 2 = 0\)
14. \(y^2 + 5y - 3 = 0\)
15. \(x^2 + 3x + 5 = 0\)
16. \(x^2 + x + 1 = 0\)
17. \(x^2 - 7x + \frac{11}{2} = 0\)
18. \(x^2 - 9x + \frac{3}{2} = 0\)
19. \(t^2 - \frac{1}{2}t - 1 = 0\)
20. \(t^2 - \frac{1}{3}t - 2 = 0\)
21. \(4x^2 - 8x - 1 = 0\)
22. \(2x^2 - 4x - 3 = 0\)
23. \(3x^2 + 6x + 1 = 0\)
24. \((5x^2 + 10x + 2 = 0)\)  
25. \((3x^2 + 2x - 3 = 0)\)  
26. \((5x^2 + 2x - 5 = 0)\)  
27. \((4x^2 - 12x - 15 = 0)\)  
28. \((2x^2 - 4x + 10 = 0)\)  
29. \((6x^2 - 24x + 42 = 0)\)  
30. \((2x^2 - x - 2 = 0)\)  
31. \((2x^2 + 3x - 1 = 0)\)  
32. \((3u^2 + 2u - 2 = 0)\)  
33. \((3u^2 - u - 1 = 0)\)  
34. \((x^2 - 4x - 1 = 15)\)  
35. \((x^2 - 12x + 8 = -10)\)  
36. \((x^2 - 12x + 8 = -10)\)  
37. \((x(x+1) - 11(x-2) = 0)\)  
38. \(((x+1)(x+7) - 4(3x+2) = 0)\)  
39. \((y^2 = (2y+3)(y-1) - 2(y-1))\)  
40. \(((2y+5)(y-5) - y(y-8) = -24)\)  
41. \(((t+2)^2 = 3(3t+1))\)  
42. \(((3t+2)(t-4) - (t-8) = 1-10t)\)  

**Answer**

1. \((-3 \pm \sqrt{10})\)  
3. \((-2 \pm \sqrt{2})\)  
5. \((-1 \pm \sqrt{3})\)  
7. \((-15, 5)\)  
9. \((-\frac{1}{3}, 1)\)  
11. \((-1 \pm \sqrt{5})\)  
13. \((-3 \pm \sqrt{17})\)  
15. \((-\frac{3}{2} \pm \frac{\sqrt{11}}{2} i)\)  
17. \((-\frac{3}{2} \pm \frac{\sqrt{11}}{2} i)\)  
19. \((-\frac{3}{2} \pm \frac{\sqrt{11}}{2} i)\)
Exercise \( \PageIndex{10} \)

Solve by completing the square and round the solutions to the nearest hundredth.

1. \((2x-1)^2=2x\)
2. \((3x-2)^2=5-15x\)
3. \((2x+1)(3x+1)=9x+4\)
4. \((3x+1)(4x-1)=17x-4\)
5. \((9x-1)-2(2x-1)=-4\)
6. \((6x+1)^2-6(6x+1)=0\)

**Answer**

1. \((0.19, 1.31)\)
2. \((-0.45, 1.12)\)
3. \((0.33, 0.67)\)

Exercise \( \PageIndex{11} \)

1. Create an equation of your own that can be solved by extracting the roots. Share it, along with the solution, on the discussion board.
2. Explain why the technique of extracting roots greatly expands our ability to solve quadratic equations.

3. Explain why the technique for completing the square described in this section requires that the leading coefficient be equal to \(1\).

4. Derive a formula for the diagonal of a square in terms of its sides.

**Answer**

1. Answer may vary

3. Answer may vary

**Footnotes**

1. Any quadratic equation in the form \(ax^2 + bx + c = 0\), where \(a, b,\) and \(c\) are real numbers and \(a \neq 0\).

2. For any real number \(k\), if \(x^2 = k\), then \(x = \pm \sqrt{k}\).

3. Applying the square root property as a means of solving a quadratic equation.

4. The process of rewriting a quadratic equation to be in the form \((x - p)^2 = q\).