UCD Mat 67: Linear Algebra

This class may well be one of your first mathematics classes that bridges the gap between the mainly computation-oriented lower division classes and the abstract mathematics encountered in more advanced mathematics courses. The goal of this class is threefold:

1. You will learn Linear Algebra, which is one of the most widely used mathematical theories around. Linear Algebra finds applications in virtually every area of mathematics, including Multivariate Calculus, Differential Equations, and Probability Theory. It is also widely applied in fields like physics, chemistry, economics, psychology, and engineering. You are even relying on methods from Linear Algebra every time you use an Internet search like Google, the Global Positioning System (GPS), or a cellphone.

2. You will acquire computational skills to solve linear systems of equations, perform operations on matrices, calculate eigenvalues, and find determinants of matrices.

3. In the setting of Linear Algebra, you will be introduced to abstraction. We will develop the theory of Linear Algebra together, and you will learn to write proofs.

The lectures will mainly develop the theory of Linear Algebra, and the discussion sessions will focus on the computational aspects. The lectures and the discussion sections go hand in hand, and it is important that you attend both. The exercises for each Chapter are divided into more computation-oriented exercises and exercises that focus on proof-writing.

As an Introduction to Abstract Mathematics is an introductory textbook designed for undergraduate mathematics majors with an emphasis on abstraction and in particular the concept of proofs in the setting of linear algebra. Typically such a student will have taken calculus, though the only prerequisite is suitable mathematical maturity. The purpose of this book is to bridge the gap between the more conceptual and computational oriented lower division undergraduate classes to the more abstract oriented upper division classes. The book begins with systems of linear equations and complex numbers, then relates these to the abstract notion of linear maps on finite-dimensional vector spaces, and covers diagonalization, eigenspaces, determinants, and the Spectral Theorem. Each chapter concludes with both proof-writing and computational exercises.
1: What is linear algebra

• 2: Introduction to Complex Numbers

\[ g(x) = \int_{a}^{x} f(t) \, dt \]

• 3. The fundamental theorem of algebra and factoring polynomials

• 4. Vector spaces
5: Span and Bases

6. Linear Maps

\[ T = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} \]

7: Eigenvalues and Eigenvectors

8. Permutations and the Determinant

9. Inner product spaces
10. Change of bases

\[ A = \int_{\sigma(A)} \lambda dE_\lambda \]

- 11. The Spectral Theorem for normal linear maps

\[
\begin{pmatrix}
a_{11} & a_{12} \\
. & . \\
a_{31} & a_{32} \\
. & .
\end{pmatrix}
\text{4×2 matrix}
\begin{pmatrix}
. & b_{12} & b_{13} \\
. & b_{22} & b_{23}
\end{pmatrix}
\text{2×3 matrix}

- 12. Supplementary notes on matrices and linear systems

- 13. Appendices

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Both hardbound and softbound versions of this textbook are available online at WorldScientific.com.