Find the Volume and Surface Area of Spheres

A sphere is the shape of a basketball, like a three-dimensional circle. Just like a circle, the size of a sphere is determined by its radius, which is the distance from the center of the sphere to any point on its surface. The formulas for the volume and surface area of a sphere are given below.

Showing where these formulas come from, like we did for a rectangular solid, is beyond the scope of this course. We will approximate $\pi$ with 3.14.

Definition: Volume and Surface Area of a Sphere

For a sphere with radius $r$:

$$ V = \frac{4}{3} \pi r^3 $$
$$ S = 4 \pi r^2 $$

Example:

A sphere has a radius 6 inches. Find its (a) volume and (b) surface area.

Solution
Step 1 is the same for both (a) and (b), so we will show it just once.

Step 1. **Read** the problem. Draw the figure and label it with the given information.

(a)

Step 2. **Identify** what you are looking for.

Step 3. **Name**. Choose a variable to represent it.

Step 4. **Translate**. Write the appropriate formula.

$$V = \frac{4}{3} \pi r^3 \tag{9.6.12}$$

Step 5. **Solve**.

$$\begin{split} V &\approx \frac{4}{3} (3.14) 6^3 \\ V &\approx 904.32 \text{ cubic inches} \end{split}$$

Step 6. **Check**.

Double-check your math on a calculator.

Step 7. **Answer** the question.

The volume is approximately 904.32 cubic inches.

(b)

Step 2. **Identify** what you are looking for.

Step 3. **Name**. Choose a variable to represent it.

Step 4. **Translate**. Write the appropriate formula.

$$S = 4 \pi r^2 \tag{9.6.13}$$

Step 5. **Solve**.

$$\begin{split} S &\approx 4 (3.14) 6^2 \\ S &\approx 452.16 \text{ sq. inches} \end{split}$$

Step 6. **Check**.

Double-check your math on a calculator.

Step 7. **Answer** the question.

The surface area is approximately 452.16 square inches.

Exercise \(\PageIndex{9}\):
Find the (a) volume and (b) surface area of a sphere with radius 3 centimeters.

**Answer a**

113.04 cu. cm

**Answer b**

113.04 sq. cm

Exercise \(\PageIndex{10}\):

Find the (a) volume and (b) surface area of each sphere with a radius of 1 foot

**Answer a**

4.19 cu. ft

**Answer b**

12.56 sq. ft

Example \(\PageIndex{6}\):

A globe of Earth is in the shape of a sphere with radius 14 centimeters. Find its (a) volume and (b) surface area. Round the answer to the nearest hundredth.

Step 1. **Read** the problem. Draw the figure and label it with the given information.

(a) 

Step 2. **Identify** what you are looking for.

the volume of the sphere

Step 3. **Name**. Choose a variable to represent it.

let \(V\) = volume

Step 4. **Translate**. Write the appropriate formula. Substitute. (Use 3.14 for \(\pi\))

\[
\begin{split}
V &= \frac{4}{3} \pi r^3 \\
V &\approx \frac{4}{3} (3.14) 14^3 \\
\end{split}
\]

```latex
\begin{split}
V &= \frac{4}{3} \pi r^3 \\
V &\approx \frac{4}{3} (3.14) 14^3 \\
&\approx 14,337.15 \\
\end{split}
```

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Step 5. Solve. \[ V \approx 11,488.21 \tag{9.6.14} \]

Step 6. Check. We leave it to you to check your calculations.

Step 7. Answer the question. The volume is approximately 11,488.21 cubic inches.

---

(b)

Step 2. Identify what you are looking for. the surface area of the sphere

Step 3. Name. Choose a variable to represent it. let S = surface area

Step 4. Translate. Write the appropriate formula. \[
\begin{split}
S &= 4 \pi r^2 \\
S &\approx 4(3.14) 14^2
\end{split}
\]

Step 5. Solve. \[ S \approx 2461.76 \tag{9.6.15} \]

Step 6. Check. We leave it to you to check your calculations.

Step 7. Answer the question. The surface area is approximately 2461.76 square inches.

---

Exercise (PageIndex{11}):

A beach ball is in the shape of a sphere with radius of 9 inches. Find its (a) volume and (b) surface area.

**Answer a**

3052.08 cu. in

**Answer b**

1017.36 sq. in.

---

Exercise (PageIndex{12}):

A Roman statue depicts Atlas holding a globe with radius of 1.5 feet. Find the (a) volume and (b) surface area of the globe.

**Answer a**

14.13 cu. ft

**Answer b**

28.26 sq. ft
Find the Volume and Surface Area of a Cylinder

If you have ever seen a can of soda, you know what a cylinder looks like. A cylinder is a solid figure with two parallel circles of the same size at the top and bottom. The top and bottom of a cylinder are called the bases. The height h of a cylinder is the distance between the two bases. For all the cylinders we will work with here, the sides and the height, h, will be perpendicular to the bases.

![Diagram of a cylinder with labels](r, radius and h, height)

Figure 5 - A cylinder has two circular bases of equal size. The height is the distance between the bases.

Rectangular solids and cylinders are somewhat similar because they both have two bases and a height. The formula for the volume of a rectangular solid, \( V = Bh \), can also be used to find the volume of a cylinder.

For the rectangular solid, the area of the base, B, is the area of the rectangular base, length \( \times \) width. For a cylinder, the area of the base, B, is the area of its circular base, \( \pi r^2 \). Figure 6 compares how the formula \( V = Bh \) is used for rectangular solids and cylinders.

![Diagram comparing rectangular solid and cylinder](b)

\[
\begin{align*}
V &= Bh \\
V &= \text{Base} \times h \\
V &= (lw) \times h \\
V &= lwh \\
V &= Bh \\
V &= \text{Base} \times h \\
V &= (\pi r^2) \times h \\
V &= \pi r^2 h
\end{align*}
\]

Figure 6 - Seeing how a cylinder is similar to a rectangular solid may make it easier to understand the formula for the volume of a cylinder.

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To understand the formula for the surface area of a cylinder, think of a can of vegetables. It has three surfaces: the top, the bottom, and the piece that forms the sides of the can. If you carefully cut the label off the side of the can and unroll it, you will see that it is a rectangle. See Figure \(\PageIndex{7}\).

![Diagram of a cylinder and unrolled label](image)

Figure \(\PageIndex{7}\) - By cutting and unrolling the label of a can of vegetables, we can see that the surface of a cylinder is a rectangle. The length of the rectangle is the circumference of the cylinder’s base, and the width is the height of the cylinder.

The distance around the edge of the can is the **circumference** of the cylinder’s base it is also the length \(L\) of the rectangular label. The height of the cylinder is the width \(W\) of the rectangular label. So the area of the label can be represented as

\[
A = L \cdot W
\]

To find the total surface area of the cylinder, we add the areas of the two circles to the area of the rectangle.

\[
S = A_{\text{top circle}} + A_{\text{bottom circle}} + A_{\text{rectangle}}
\]

\[
S = \pi r^2 + \pi r^2 + 2\pi r \cdot h
\]

\[
S = 2\pi r^2 + 2\pi rh
\]

The surface area of a cylinder with radius \(r\) and height \(h\), is

\[
S = 2\pi r^2 + 2\pi rh
\]

**Definition: Volume and Surface Area of a Cylinder**

For a cylinder with radius \(r\) and height \(h\):
Example \((\PageIndex{7})\):

A cylinder has height 5 centimeters and radius 3 centimeters. Find the (a) volume and (b) surface area.

**Solution**

Step 1. **Read** the problem. Draw the figure and label it with the given information.

(a)

Step 2. **Identify** what you are looking for.

Step 3. **Name**. Choose a variable to represent it.

Step 4. **Translate**. Write the appropriate formula. Substitute. (Use 3.14 for \(\pi\))

\[
\begin{split}
V &= \pi r^2 h \\
V &\approx (3.14) 3^2 \cdot 5
\end{split}
\]

Step 5. **Solve**.

\[
V \approx 141.3 \tag{9.6.17}
\]

Step 6. **Check**.

We leave it to you to check your calculations.

Step 7. **Answer** the question.

The volume is approximately 141.3 cubic inches.

(b)

Step 2. **Identify** what you are looking for.

the surface area of the cylinder

Step 3. **Name**. Choose a variable to represent it.

Step 4. **Translate**. Write the appropriate formula.

\[
S = 2\pi r^2 + 2\pi rh
\]

Step 5. **Solve**.

\[
S \approx 2 \cdot 3.14 \cdot 3^2 + 2 \cdot 3.14 \cdot 3 \cdot 5
\]

Step 6. **Check**.

We leave it to you to check your calculations.

Step 7. **Answer** the question.

The surface area is approximately 94.2 square inches.
Step 3. **Name.** Choose a variable to represent it.

**let S = surface area**

Step 4. **Translate.** Write the appropriate formula.

\[
\begin{split}
S &= 2 \pi r^2 + 2 \pi rh \\
\approx & 2(3.14)3^2 + 2(3.14)(3)5 \\
\end{split}
\]

Step 5. **Solve.**

\[
S \approx 150.72 \tag{9.6.18}
\]

Step 6. **Check.**

We leave it to you to check your calculations.

Step 7. **Answer the question.**

The surface area is approximately 150.72 square inches.

---

Exercise \(\PageIndex{13}\):

Find the (a) volume and (b) surface area of the cylinder with radius 4 cm and height 7 cm.

**Answer a**

351.68 cu. cm

**Answer b**

276.32 sq. ft

---

Exercise \(\PageIndex{14}\):

Find the (a) volume and (b) surface area of the cylinder with given radius 2 ft and height 8 ft.

**Answer a**

100.48 cu. ft

**Answer b**

125.6 sq. ft

---

Example \(\PageIndex{8}\):

Find the (a) volume and (b) surface area of a can of soda. The radius of the base is 4 centimeters and the height is 13 centimeters. Assume the can is shaped exactly like a cylinder.

**Solution**
Step 1. **Read** the problem. Draw the figure and label it with the given information.

(a) **Step 2. Identify** what you are looking for. the volume of the cylinder

**Step 3. Name.** Choose a variable to represent it. let V = volume

**Step 4. Translate.** Write the appropriate formula. Substitute. (Use 3.14 for \(\pi\))

\[
\begin{align*}
V &= \pi r^2 h \\
V &\approx (3.14) 4^2 \cdot 13
\end{align*}
\]

**Step 5. Solve.**

\[
V \approx 653.12 \tag{9.6.19}
\]

**Step 6. Check.** We leave it to you to check.

**Step 7. Answer** the question. The volume is approximately 653.12 cubic centimeters.

(b) **Step 2. Identify** what you are looking for. the surface area of the cylinder

**Step 3. Name.** Choose a variable to represent it. let S = surface area

**Step 4. Translate.** Write the appropriate formula.

\[
\begin{align*}
S &= 2 \pi r^2 + 2 \pi rh \\
S &\approx 2(3.14)4^2 + 2(3.14)(4)13
\end{align*}
\]

**Step 5. Solve.**

\[
S \approx 427.04 \tag{9.6.20}
\]

**Step 6. Check.** We leave it to you to check your calculations.

**Step 7. Answer** the question. The surface area is approximately 427.04 square centimeters.

Exercise (PageIndex{15}): Find the (a) volume and (b) surface area of a can of paint with radius 8 centimeters and height 19 centimeters. Assume the can is shaped exactly like a cylinder.
Find the Volume of Cones

The first image that many of us have when we hear the word ‘cone’ is an ice cream cone. There are many other applications of cones (but most are not as tasty as ice cream cones). In this section, we will see how to find the volume of a cone.

In geometry, a cone is a solid figure with one circular base and a vertex. The height of a cone is the distance between its base and the vertex. The cones that we will look at in this section will always have the height perpendicular to the base. See Figure \(\PageIndex{8}\).

![Figure \(\PageIndex{8}\) - The height of a cone is the distance between its base and the vertex.](image)

Earlier in this section, we saw that the volume of a cylinder is \(V = \pi r^2 h\). We can think of a cone as part of a cylinder. Figure \(\PageIndex{9}\) shows a cone placed inside a cylinder with the same height and same base. If we compare the volume of the cone and the cylinder, we can see that the volume of the cone is less than that of the cylinder.
Figure \( \PageIndex{9} \) - The volume of a cone is less than the volume of a cylinder with the same base and height.

In fact, the volume of a cone is exactly one-third of the volume of a cylinder with the same base and height. The volume of a cone is

$$ V = \frac{1}{3} \pi r^2 h \tag{9.6.21} $$

Since the base of a cone is a circle, we can substitute the formula of area of a circle, \( \pi r^2 \), for \( B \) to get the formula for volume of a cone.

$$ V = \frac{1}{3} \pi r^2 h \tag{9.6.22} $$

In this book, we will only find the volume of a cone, and not its surface area.

Definition: Volume of a Cone

For a cone with radius \( r \) and height \( h \).

Example \( \PageIndex{9} \):  

Find the volume of a cone with height 6 inches and radius of its base 2 inches.

Solution
Step 1. **Read** the problem. Draw the figure and label it with the given information.

![Diagram of a cone](image)

Step 2. **Identify** what you are looking for.

- the volume of the cone

Step 3. **Name**. Choose a variable to represent it.

- let \( V = \text{volume} \)

Step 4. **Translate**. Write the appropriate formula. Substitute. (Use \( 3.14 \) for \( \pi \))

\[
\begin{align*}
V &= \frac{1}{3} \pi r^2 \cdot h \\
V &\approx \frac{1}{3} \times 3.14 \times (2)^2 \times 6
\end{align*}
\]

Step 5. **Solve**.

\[
V \approx 25.12 \tag{9.6.23}
\]

Step 6. **Check**.

- We leave it to you to check your calculations.

Step 7. **Answer** the question.

- The volume is approximately 25.12 cubic inches.

Exercise \(\PageIndex{17}\):

Find the volume of a cone with height 7 inches and radius 3 inches

**Answer**

- 65.94 cu. in.

Exercise \(\PageIndex{18}\):

Find the volume of a cone with height 9 centimeters and radius 5 centimeters

**Answer**

- 235.5 cu. cm

Example \(\PageIndex{10}\):

Marty’s favorite gastro pub serves French fries in a paper wrap shaped like a cone. What is the volume of a conic wrap that is 8 inches tall and 5 inches in diameter? Round the answer to the nearest hundredth.

**Solution**
Step 1. **Read** the problem. Draw the figure and label it with the given information.

Step 2. **Identify** what you are looking for.

Step 3. **Name**. Choose a variable to represent it.

Step 4. **Translate**. Write the appropriate formula. Substitute. (Use 3.14 for $\pi$, and notice that we were given the distance across the circle, which is its diameter. The radius is 2.5 inches.)

$$
V = \frac{1}{3} \pi r^2 h \\
V \approx \frac{1}{3} (3.14)(2.5)^2 (8)
$$

Step 5. **Solve**.

Step 6. **Check**.

Step 7. **Answer** the question.

Exercise \(\PageIndex{19}\):

How many cubic inches of candy will fit in a cone-shaped piñata that is 18 inches long and 12 inches across its base? Round the answer to the nearest hundredth.

**Answer**

678.24 cu. in.

Exercise \(\PageIndex{20}\):

What is the volume of a cone-shaped party hat that is 10 inches tall and 7 inches across at the base? Round the answer to the nearest hundredth.

**Answer**

128.2 cu. in.
Summary of Geometry Formulas

The following charts summarize all of the formulas covered in this chapter.

### Supplementary and Complementary Angles
- $m\angle A + m\angle B = 180^\circ$ for supplementary angles $A$ and $B$
- $m\angle C + m\angle D = 90^\circ$ for complementary angles $C$ and $D$

### Triangle
- For $\triangle ABC$, angle measures.
- $m\angle A + m\angle B + m\angle C = 180^\circ$
- Perimeter, $P = a + b + c$
- $A = \frac{1}{2}bh$

### Similar Triangles
- $\frac{m\angle A}{m\angle X} = \frac{m\angle B}{m\angle Y} = \frac{m\angle C}{m\angle Z}$
- $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

### Sphere
- Volume: $V = \frac{4}{3}\pi r^3$
- Surface Area: $S = 4\pi r^2$

### Cylinder
- Volume: $V = \pi r^2h$ or $V = Bh$
- Surface Area: $S = 2\pi r^2 + 2\pi rh$

### Circle
- Circumference: $C = 2\pi r$
- Area: $A = \pi r^2$

### Cone
- $V = \frac{1}{3}\pi r^2h$

ACCESS ADDITIONAL ONLINE RESOURCES

Volume of a Cone

**Practice Makes Perfect**

**Find Volume and Surface Area of Rectangular Solids**

In the following exercises, find (a) the volume and (b) the surface area of the rectangular solid with the given dimensions.

263. length 2 meters, width 1.5 meters, height 3 meters
264. length 5 feet, width 8 feet, height 2.5 feet
265. length 3.5 yards, width 2.1 yards, height 2.4 yards
266. length 8.8 centimeters, width 6.5 centimeters, height 4.2 centimeters

In the following exercises, solve.
267. **Moving van** A rectangular moving van has length 16 feet, width 8 feet, and height 8 feet. Find its (a) volume and (b) surface area.

268. **Gift box** A rectangular gift box has length 26 inches, width 16 inches, and height 4 inches. Find its (a) volume and (b) surface area.

269. **Carton** A rectangular carton has length 21.3 cm, width 24.2 cm, and height 6.5 cm. Find its (a) volume and (b) surface area.

270. **Shipping container** A rectangular shipping container has length 22.8 feet, width 8.5 feet, and height 8.2 feet. Find its (a) volume and (b) surface area.

In the following exercises, find (a) the volume and (b) the surface area of the cube with the given side length.

271. 5 centimeters

272. 6 inches

273. 10.4 feet

274. 12.5 meters

In the following exercises, solve.

275. **Science center** Each side of the cube at the Discovery Science Center in Santa Ana is 64 feet long. Find its (a) volume and (b) surface area.

276. **Museum** A cube-shaped museum has sides 45 meters long. Find its (a) volume and (b) surface area.

277. **Base of statue** The base of a statue is a cube with sides 2.8 meters long. Find its (a) volume and (b) surface area.

278. **Tissue box** A box of tissues is a cube with sides 4.5 inches long. Find its (a) volume and (b) surface area.

---

**Find the Volume and Surface Area of Spheres**

In the following exercises, find (a) the volume and (b) the surface area of the sphere with the given radius. Round answers to the nearest hundredth.

279. 3 centimeters

280. 9 inches

281. 7.5 feet

282. 2.1 yards

In the following exercises, solve. Round answers to the nearest hundredth.

283. **Exercise ball** An exercise ball has a radius of 15 inches. Find its (a) volume and (b) surface area.

284. **Balloon ride** The Great Park Balloon is a big orange sphere with a radius of 36 feet. Find its (a) volume and (b) surface area.

285. **Golf ball** A golf ball has a radius of 4.5 centimeters. Find its (a) volume and (b) surface area.

286. **Baseball** A baseball has a radius of 2.9 inches. Find its (a) volume and (b) surface area.
Find the Volume and Surface Area of a Cylinder

In the following exercises, find (a) the volume and (b) the surface area of the cylinder with the given radius and height. Round answers to the nearest hundredth.

287. radius 3 feet, height 9 feet
288. radius 5 centimeters, height 15 centimeters
289. radius 1.5 meters, height 4.2 meters
290. radius 1.3 yards, height 2.8 yards

In the following exercises, solve. Round answers to the nearest hundredth.

291. Coffee can A can of coffee has a radius of 5 cm and a height of 13 cm. Find its (a) volume and (b) surface area.
292. Snack pack A snack pack of cookies is shaped like a cylinder with radius 4 cm and height 3 cm. Find its (a) volume and (b) surface area.
293. Barber shop pole A cylindrical barber shop pole has a diameter of 6 inches and height of 24 inches. Find its (a) volume and (b) surface area.
294. Architecture A cylindrical column has a diameter of 8 feet and a height of 28 feet. Find its (a) volume and (b) surface area.

Find the Volume of Cones

In the following exercises, find the volume of the cone with the given dimensions. Round answers to the nearest hundredth.

295. height 9 feet and radius 2 feet
296. height 8 inches and radius 6 inches
297. height 12.4 centimeters and radius 5 cm
298. height 15.2 meters and radius 4 meters

In the following exercises, solve. Round answers to the nearest hundredth.

299. Teepee What is the volume of a cone-shaped teepee tent that is 10 feet tall and 10 feet across at the base?
300. Popcorn cup What is the volume of a cone-shaped popcorn cup that is 8 inches tall and 6 inches across at the base?
301. Silo What is the volume of a cone-shaped silo that is 50 feet tall and 70 feet across at the base?
302. Sand pile What is the volume of a cone-shaped pile of sand that is 12 meters tall and 30 meters across at the base?

Everyday Math

303. Street light post The post of a street light is shaped like a truncated cone, as shown in the picture below. It is a large cone minus a smaller top cone. The large cone is 30 feet tall with base radius 1 foot. The smaller cone is 10 feet tall with base radius of 0.5 feet. To the nearest tenth, (a) find the volume of the large cone. (b) find the volume
of the small cone. (c) find the volume of the post by subtracting the volume of the small cone from the volume of the large cone.

304. **Ice cream cones** A regular ice cream cone is 4 inches tall and has a diameter of 2.5 inches. A waffle cone is 7 inches tall and has a diameter of 3.25 inches. To the nearest hundredth, (a) find the volume of the regular ice cream cone. (b) find the volume of the waffle cone. (c) how much more ice cream fits in the waffle cone compared to the regular cone?

305. The formulas for the volume of a cylinder and a cone are similar. Explain how you can remember which formula goes with which shape.

306. Which has a larger volume, a cube of sides of 8 feet or a sphere with a diameter of 8 feet? Explain your reasoning.

**Self Check**

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

<table>
<thead>
<tr>
<th>I can...</th>
<th>Confidently</th>
<th>With some help</th>
<th>No-I don't get it!</th>
</tr>
</thead>
<tbody>
<tr>
<td>find volume and surface area of rectangular solids.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>find volume and surface area of spheres.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>find volume and surface area of cylinders.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>find volume of cones.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) After reviewing this checklist, what will you do to become confident for all objectives?

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