10.4: Bessel Functions of General Order

The recurrence relation for the Bessel function of general order \(\pm \nu\) can now be solved by using the gamma function,

\[
[a_m = -\frac{1}{m(m\pm 2\nu)} a_{m-2}]
\]

has the solutions \((x > 0)\)

\[
\begin{aligned}
J_{\nu}(x) &= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\Gamma(\nu+k+1)}
\left(\frac{x}{2}\right)^{\nu+2k}, \\
J_{-\nu}(x) &= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\Gamma(-\nu+k+1)}
\left(\frac{x}{2}\right)^{-\nu+2k}.
\end{aligned}
\]

The general solution to Bessel’s equation of order \(\nu\) is thus

\[
y(x) = A J_{\nu}(x)+BJ_{-\nu}(x),
\]

for any non-integer value of \(\nu\). This also holds for half-integer values (no logs).